Homework 11

due Dec 2, 2014

In addition to

4.3. 2, 4, 6, 8, 10, 12, 14, 17-22, 32, 33, 36, 37

4.4. 1-11, 13, 17, 18, 22, 24, 32, 37, 38

complete the following problems.

1. A differential equation is an equation in which some function is related to its own derivative(s). For each of the following functions, calculate the appropriate derivative, and show that the function satisfies the indicated differential equation.

(a) $f(x) = 2e^{-3x}$, $f'(x) = -3f(x)$.

(b) $f(t) = -e^{kt}$, $f'(t) = kf(t)$.

(c) $f(t) = 1 - e^{-t}$, $f'(t) = 1 - f(t)$.

2. Consider the function $y = f(t) = Ce^{kt}$ where C and k are constants. For what value(s) of these constants does this function satisfy the equation

(a) $\frac{dy}{dt} = -5y$.

(b) $\frac{dy}{dt} = 3y$.

3. Two populations. Two populations are studied. Population 1 is found to obey the differential equation

$$\frac{dy_1}{dt} = 0.2y_1$$

and population 2 obeys

$$\frac{dy_2}{dt} = -0.3y_2,$$

where t is time in years.

(a) Which population is growing and which is declining?

(b) Find the doubling time (respectively half-life) associated with the given population.

(c) If the initial levels of the two populations were $y_1(0) = 100$ and $y_2(0) = 10000$, how big would each population be at arbitrary time t?

(d) At what time would the two populations be exactly equal?

4. Rodent population. The per capita birthrate of one species of rodent is 0.05 newborns per day. (This means that, on average, each member of the population will result in 5 newborn rodents every 100 days.) Suppose that over the period of 1000 days there are no deaths, and that the initial population of rodents is 250. Write a differential equation for the population size $N(t)$ at time t (in days). Write down the initial condition that N satisfies. Find the solution, i.e. express N as some function of time t that satisfies your differential equation and initial condition. How many rodents will there be after 1 year ?