Homework 5

Additional problems

due September 30, 2014

In addition to

2.7: 2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 90, 94, 96, 97, 98, 100, 102, 104

2.8: 2, 8, 14, 20, 26, 32, 38, 44, 50, 56, 62, 70, 72, 74, 76, 78, 80, 82, 84

complete the following problems.

Note: There are a LOT more problems in 2.7 and 2.8. I recommend that you take advantage and do as many practice problems as possible! Finding the derivative is a technical skill, and like any other, to get good you have to PRACTICE, even if you've seen this material before.

1. Use the Intermediate Value Theorem to show that the function

$$f(x) = x^3 - x^2 + x + 2$$

has a real root in the interval $[-1, 0]$.

2. Give an example to show that the conclusion of the Intermediate Value Theorem fails if the given function $f(x)$ is not required to be continuous on the interval $a \leq x \leq b$.

3. Logistic growth rate. In logistic growth, the rate of growth of a population, R depends on the population size N as follows:

$$R = rN\left(1 - \frac{N}{K}\right),$$

where r and K are positive constants.

(a) Find the rate of change of the growth rate with respect to the population size, $\frac{dR}{dN}$.

(b) At what population size(s) is the growth rate 0? Interpret these population size(s).

(c) What is $\frac{dR}{dN}$ at those population sizes? Give an interpretation.

4. Convergent extension. Most animals are longer head to tail than side to side. To obtain relative elongation along one axis, the embryo undergoes a process called "convergent extension" whereby a block of tissue elongates (extends) along one axis and narrows (converges) along the other axis as shown in the figure below. Here we consider this process. Suppose a block of tissue originally having dimensions length L = width w = 10mm, and thickness τ = 1 mm, extends in length at the rate of 1 mm per day, while the volume V and thickness τ remain fixed. At what rate is the width w of the tissue block changing when the length is L = 20 mm? Assume the tissue is a rectangular block (volume = length$\times$width$\times$thickness).