Final Exam Review

Terms to Know:

Listed at the start of each chapter "Summary and Review":

Chapter 2: page 155

Chapter 3: page 257

Chapter 4: page 330

Chapter 5: page 422 (up to Average Value only).

Exercises:

These are intended to supplement your studying. Use as a guide, to identify weak areas to work on. The "additional problems" **by no means** cover all the material that is fair game for the final!

Don't forget: Practice taking limits, computing derivatives, and integrating - these should come automatically.

Problems from the textbook

Chapter 2 review (pages 155-156): 5, 7, 9, 10-15, 17, 19, 21, 23, 25, 28, 29, 31, 33, 37, 39, 43

Chapter 3 review (pages 257-258): 1, 9, 11, 13, 15, 23, 25, 32, 33, 35

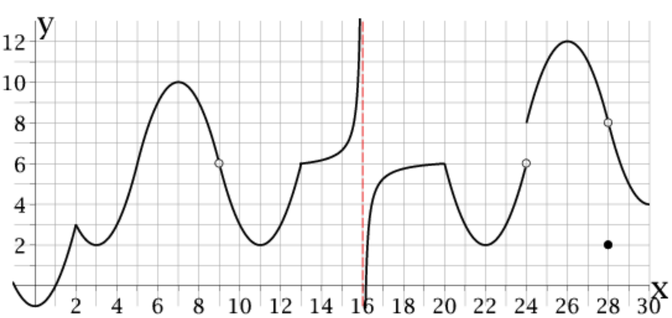
Chapter 4 review (pages 330-331): 1, 3, 5, 7, 9, 11, 19, 21, 25

Chapter 5 review (pages 422-423): 1-11, 13, 15, 16

Note: Most answers are at the back of the book.

Additional problems

1. Let $f(x)$ be a function with graph $y = f(x)$ shown below:



(a) Find and classify all points on the interval $[0,30]$ at which f is discontinuous.

(b) Find all points on the interval $[0,30]$ where f is not differentiable.

2. A ladder 5 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate of 0.5 m/min, (a) how fast is the top of the ladder sliding down the wall when the base of the ladder is 1 m away from the wall? (b) how fast is the slope of the ladder changing when the base of the ladder is 1 m away from the wall?

3. Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume ?

4. Find the equation of the line tangent to the function $f(x) = e^x \cos(x)$ when $x = 0$. Please be sure to write your final answer in the form $y = mx + b$.

5. (a) Find the derivative dy/dx of the curve $\sin(x + y) = xy$.

(b) Use the result from part (a) above to find the equation of the line tangent to the curve $\sin(x + y) = xy$ at the point $(0,0)$.

6. If $V'(t)$ is the rate at which the water flows into a reservoir at time t, what does $\int_{t_1}^{t_2} V'(t)\,dt$ represent?

7. A population of animals has a per-capita birth rate of $b = 0.08$ per year and a per-capita death rate of $m = 0.01$ per year. The population density, $P(t)$ is found to satisfy the differential equation

$$\frac{dP}{dt} = bP(t) - mP(t)$$

(a) If the population is initially $P(0) = 1000$, find how big the population will be in 5 years.

(b) When will the population double?

8. (a) Find the solution to the initial value problem $\frac{dP}{dt} = 0.5P$, $P(0) = 2$.

(b) Write $P(t) = 3e^{-t}$ as an initial value problem.

9. Find the area enclosed by the curves $y = 2x^2 - 6x + 5$ and $y = x^2 + 6x - 15$.

10. Let $f(x) = 1/x$.

(a) Approximate the area under the curve $y = f(x)$ on the interval $[1,4]$ using a Riemann sum with 3 subintervals.

(b) Set up and evaluate a definite integral that gives the exact area under the curve $f(x)$ on the interval $[1,4]$.

ANSWERS:

1. (a) $f(x)$ not continuous at $x = 9$ (hole or removable discontinuity), $x = 16$ (infinite discontinuity), $x = 24$ (jump discontinuity), $x = 28$ (hole or removable discontinuity). (b) $f(x)$ is not differentiable at $x = 9$, 16, 24, 28, where f is not continuous, and also not differentiable at $x = 2$, 13, 20 (corners).

2. (a) $-1/4\sqrt{6}$ m/s. (b) $25/4\sqrt{6}\ \mathrm{s}^{-1}$.

3. 4 inches $\times$ 2 inches, rotating around the edge of length 2 inches.

4. $f'(x) = e^x(\cos(x) - \sin(x))$ so slope $m = f'(0) = 1$. Tangent line equation is $y = x + 1$.

5. (a) $dy/dx = (y - \cos(x + y))/(\cos(x + y) - x)$. (b) $y = -x$.

6. The amount of water that flowed into the reservoir between time t_1 and t_2.

7. (a) $P(t) = 1000e^{0.07t}$, so $P(5) = 1000e^{0.35}$.

(b) Population doubles at time $T = \ln(2)/0.07$ or $100\ln(2)/7$.

8. (a) $P(t) = 2e^{0.5t}$. (b) $\frac{dP}{dt} = -P$, $P(0) = 3$.

9. Area = $\int_2^{10} \left((x^2+6x-15)-(2x^2-6x+5)\right)\, dx = 256/3$.

10. (a) $\Delta x = 1$, $x_i = 1+i$, so $\sum_{j=1}^{n} f(x_i)\Delta x = \sum_{j=1}^{3} 1/(1+i) = 1/2+1/3+1/4 = 13/12$.

(b) $\int_1^4 \frac{1}{x}\, dx = \ln|x|\big|_1^4 = \ln(4)-\ln(1) = \ln(4)$ or $2\ln(2)$.