the optimal solution we need. i.e. there are no larger values at endpoints of the interval $0 \leq h \leq 2L$.

To finish the problem, we can find the radius of the barrel having this height by plugging this result for h into the constraint equation, i.e. using

$$r^2 = \frac{1}{4}\left(L^2 - \frac{h^2}{4}\right) = \frac{1}{4}\left(L^2 - \frac{L^2}{3}\right) = \frac{1}{4}\left(\frac{2}{3}L^2\right).$$

After simplifying and rewriting, we get

$$r = \frac{1}{\sqrt{3}\sqrt{2}}L.$$

The shape of the wine barrel with largest volume for the given price can now be specified. One way to do this is to specify the ratio of height to radius. (Tall skinny barrels have a high ratio h/r and squat fat ones have a low ratio.) By the above reasoning, the ratio of h/r for the optimal barrel is

$$\frac{h}{r} = \frac{2\frac{L}{\sqrt{3}}}{\frac{1}{\sqrt{3}\sqrt{2}}L} = 2\sqrt{2}. \tag{7.5}$$

The height of the barrel should be $2\sqrt{2} \approx 3$ times the radius in these most economical wine barrels.

7.3 Checking endpoints

In some cases, the optimal value of a function will not occur at any of its local maxima, but rather at one of the endpoints of an interval. Here we consider this situation.

Section 7.3 Learning goals

1. Understand the distinction between local and global extrema.
2. Be able to find the global minimum or maximum in a given word problem.

The following example illustrates this point:

Example 7.5 (maximal perimeter) The area of a rectangle having sides of length x and y is $A = xy$. Suppose that that the variable x is only allowed to take values in the range $0.5 \leq x \leq 4$ Find the dimensions of the rectangle having largest perimeter whose area $A = 1$ is fixed. (The perimeter of a rectangle is the total length of its outer edge.) ■

Solution: The perimeter of a rectangle whose sides are length x, y is

$$P = x + y + x + y = 2x + 2y.$$

We are asked to maximize this quantity. Since the area of the rectangle is $A = xy$, and this is given, we obtain $xy = 1$ as the constraint. Using the constraint, we can solve for y.

$$y = \frac{1}{x}.$$

Then, substituting this result leads to a function depending only on x:

$$P(x) = 2x + \frac{2}{x}.$$

To find critical points, we set

$$P'(x) = 2\left(1 - \frac{1}{x^2}\right) = 0.$$

Thus, $x^2 = 1$ or $x = \pm 1$. We reject the negative root as it is irrelevant for the (positive) side length of the rectangle. Checking if this is a maximum we find that

$$P''(x) = \frac{4}{x^3} > 0$$

so we have found a local *minimum*! This is clearly not the maximum we were looking for.

We must thus check the endpoints of the interval for the maximal value of the function. We find that $P(4) = 8.5$ and $P(0.5) = 5$. The largest perimeter for the rectangle will thus occur when $x = 4$, indeed at the endpoint of the domain, as shown in Figure 7.5.

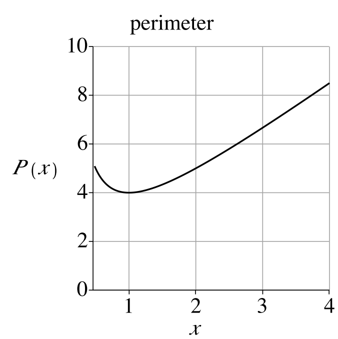

Figure 7.5. *In Example 7.5, the critical point we found is a local minimum. To maximize the perimeter of the rectangle, we must consider the end points of the interval* $0.5 \leq x \leq 4$.

7.4 Optimal foraging

Animals need to spend a considerable part of their time searching for food. There is a limited time available for this activity, since when the sun goes down, risk of becoming food