Midterm 1 Review

Concepts:

- Limits: Calculating the limit, calculating the limit from the right, calculating the limit from the left. Limit exists when left and right limits are equal to each other, that is, $\lim_{x\to c} f(x)$ exists when $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$. Then $\lim_{x\to c} f(x) = L$.
- Continuity: A function $f(x)$ is continuous at a point when the limit exists and is equal to the function at that point, i.e. $\lim_{x\to c} f(x) = f(c)$. The function is continuous on an interval when that holds for all points in the interval. Pencil test!
- Types of discontinuities: Jump, removable ("holes"), infinite.
- Average rates of change and the difference quotient, $\frac{f(x+h)-f(x)}{h}$.
- The derivative: Slope of the tangent curve. Limit of the difference quotient, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
- Computing the derivative: power rule, sum/difference rule, product rule, quotient rule, chain rule. Higher order derivatives.
- Applications of the derivative: relative extrema, inflection points, intervals of increase/decrease, concavity. First and second derivative tests.
- Graphical interpretations: continuity and derivatives.

Exercises:

Problems from the textbook

Chapter 2 review (page 155-156): 4-19 (also identify types of discontinuities), 23, 25, 26, 29, 31, 33, 38, 39, 43.

Chapter 3 review (page 257-258): 2,3,7

Note: All answers are at the back of the book.

Additional problems

1. Evaluate the following limits, if they exist: (a) $\lim_{t\to 1} \frac{t^2-1}{t-1}$. (b) $\lim_{t\to 1} \frac{t^2-1}{t+1}$. (c) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$.
2. The function
$$f(x) = \frac{x^2-9}{x-3}$$
is not defined at a certain point. Find that point. How should it be defined to make $f(x)$ continuous there?

3. Let

$$f(x) = \begin{cases} x^3, & x < -1 \\ x, & -1 < x < 1 \\ 1-x, & x \geq 1 \end{cases}$$

Find (a) $f(1)$, (b) $\lim_{x\to 1^+} f(x)$, (c) $\lim_{x\to 1^-} f(x)$, (d) $\lim_{x\to -1} f(x)$. (e) What are the values of x where $f(x)$ is discontinuous? Name the discontinuity. (f) How should f be defined at $x = -1$ to make it continuous there?

4. Find the values of a and b so that the following function is continuous everywhere.

$$f(x) = \begin{cases} x+1, & x < 1 \\ ax+b, & 1 \leq x < 2 \\ 3x, & x \geq 2 \end{cases}$$

5. Show that the equation $x^5 + 4x^3 - 7x + 14 = 0$ has at least one real solution.

6. Use the Intermediate Value Theorem to show that $x^3 + 3x - 2 = 0$ has a real solution between 0 and 1.

7. Use the limit definition of the derivative (aka the limit of the difference quotient) to find the derivative of (a) $f(x) = x^2 - 5x$, (b) $f(x) = 1/(x-3)$, (c) $f(x) = \sqrt{9-x}$.

8. Sketch (a) a function that is not continuous at $x = 3$. (b) a function that is continuous but not differentiable at $x = 3$.

9. Find the following derivatives: (a) $\frac{d}{dx}\left(x^3 - 3x^2 + x^{-2}\right)$, (b) $\frac{d}{dt}\left(t\sqrt{2t+1}\right)$, (c) $\frac{d}{dt}\left(\sin^2(\cos(4t))\right)$, (d) $f'(2)$ if $f(x) = (x^2-1)^2(3x^3-4x)$.

10. Find the coordinates of the point on the curve $f(x) = (x-2)^2$ where the tangent line is parallel to the line $y = 2x+3$. Then find the equation of that tangent line.

11. Suppose $f(2) = 3$, $f'(2) = 4$, $f''(2) = -1$, $g(2) = 2$, and $g'(2) = 5$. Find each value. (a) $\frac{d}{dx}\left[f(x)^2 + g(x)^3\right]$ at $x = 2$. (b) $\frac{d}{dx}[f(x)g(x)]$ at $x = 2$. (c) $\frac{d}{dx}[f(g(x))]$ at $x = 2$.

12. Sketch $g(x) = 3x^4 + 4x^3 - 12x^2 + 5$. Find all intervals of increase and decrease, relative maxima and minima, intervals of concavity, and inflection points.