12. Sketch $g(x) = 3x^4 + 4x^3 - 12x^2 + 5$. Find all intervals of increase and decrease, relative maxima and minima, intervals of concavity, and inflection points.

Solution: In addition to the function itself we'll need its first and second derivative. Start with those:

$$g'(x) = 12x^3 + 12x^2 - 24x$$
$$g''(x) = 36x^2 + 24x - 24$$

Critical points occur where $g'(x) = 0$. Solving,

$$12x^3 + 12x^2 - 24x = 0$$
$$12x(x^2 + x - 2) = 0$$
$$12x(x-1)(x+2) = 0.$$

The critical points are $x = 0$, $x = 1$, and $x = -2$. Since $g''(-2) = 72 > 0$, the function is locally concave up at $x = -2$ and $g(-2) = -27$ is a relative minimum. $g''(0) = -24 < 0$, the function is locally concave down at $x = 0$ and $g(0) = 5$ is a relative maximum. And $g''(1) = 36 > 0$ so the function is locally concave up at $x = 1$ and $g(1) = 0$ is a relative minimum.

Inflection points occur where $g''(x) = 0$. Solving,

$$36x^2 + 24x - 24 = 0$$
$$12\left(3x^2 + 2x - 2\right) = 0.$$

which has solutions $x = \left(1 \pm \sqrt{7}\right)/3$, using the quadratic formula. $\left(\left(1+\sqrt{7}\right)/3, f\left(\left(1+\sqrt{7}\right)/3\right)\right) = \left(\left(1+\sqrt{7}\right)/3, (-149+80\sqrt{7})/27\right) \approx (0.55, 2.32)$ and $\left(\left(1-\sqrt{7}\right)/3, f\left(\left(1-\sqrt{7}\right)/3\right)\right) = \left(\left(1-\sqrt{7}\right)/3, (-14\right.$ $(-1.22, -13.36)$.

To sketch, we'll also want $x-$intercepts, $y-$intercepts.

$x-$intercepts: where $g(x) = 0$. $g(x) = x^4 - 2x^3 = 0$ has solutions $x = 0$ and $x = 2$. These are the $x-$intercepts.

$y-$intercepts: where $x = 0$. $g(0) = 0$ is the $y-$intercept.

We can use the local behavior at the critical points to discern the intervals of increase/decrease, and concavity, and write down our answer:

Answer:

- The function $g(x)$ is decreasing for $x < -2$ and $0 < x < 1$, and increasing for $-2 < x < 0$ and $x > 1$.
- The points $(-2, 27)$ and $(1, 1)$ are relative minima, while $(0, 5)$ is a relative maxima.
- The function is concave up for $x < \left(1 - \sqrt{7}\right)/3$ and $x > \left(1 + \sqrt{7}\right)/3$, and concave down for $\left(1 - \sqrt{7}\right)/3 < x < \left(1 + \sqrt{7}\right)/3$.

- The points $\left(\left(1+\sqrt{7}\right)/3, (-149+80\sqrt{7})/27\right)$ and $\left(\left(1-\sqrt{7}\right)/3, (-149-80\sqrt{7})/27\right)$ are the inflection points.

However if you're not comfortable doing that you can always use tables (which for the first derivative, amounts to the first derivative test)!

For intervals of increase/decrease

	$x < -2$	$-2 < x < 0$	$0 < x < 1$	$x > 1$
Test point x^*	-3	-1	0.5	2
$g'(x^*)$	-144	24	-7.5	96
Behavior of $g(x)$	decreasing	increasing	decreasing	increasing

And for concavity

	$x < \left(1-\sqrt{7}\right)/3$	$\left(1-\sqrt{7}\right)/3 < x < \left(1+\sqrt{7}\right)/3$	$x > \left(1+\sqrt{7}\right)/3$
Test point x^*	-2	-0	1
$g''(x^*)$	72	-24	36
Behavior of $f(x)$	concave up	concave down	concave up

Now we can safely sketch.