Math 141H-01

PRACTICE Midterm Exam 1

February X, 2015

Name:_____

Instructions: Clearly answer each of the questions. Box your answers. Partial credit will be awarded based on the clarity and correctness of your explanation of each solution. Use the backs of pages for your scratch work. Make sure you have all 11 pages.

Trigonometric identities and useful integration formulas are given on pages 10-11.

1. (X points) Inverse Functions.
(a) Does $f(x) = x^2 - 1$ have an inverse? Why or why not?

(b) Given $g(x) = x^3 - 8$, what is $g^{-1}(-8)$?

(c) Given $h^{-1}(x) = \tan^{-1}(2x+1)$, what is $h(\pi/4)$?

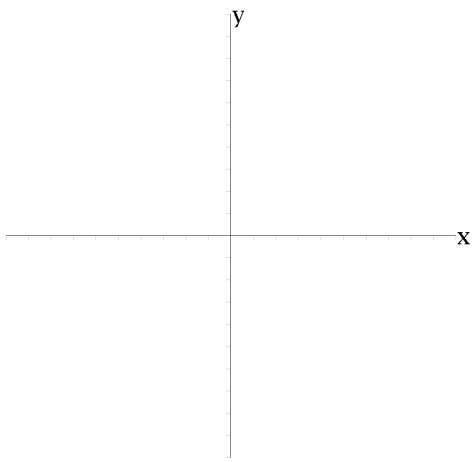
(d) Let f be a function such that

$$f(-4) = 5, \quad f(1) = 2, \quad f(2) = 1, \quad f(3) = -4, \quad f(5) = -2$$

$$f'(-4) = 3, \quad f'(-2) = 1, \quad f'(2) = \frac{1}{4}, \quad f'(3) = -\frac{1}{4}, \quad f'(5) = 2$$

Find $(f^{-1})'(-2)$ and $(f^{-1})'(5)$.

(e) $w(x) = \sqrt{x-1}$. Sketch $w^{-1}(x)$



2. (X points)

(a) What is the domain of $y = \ln(x^2 - 1)$?

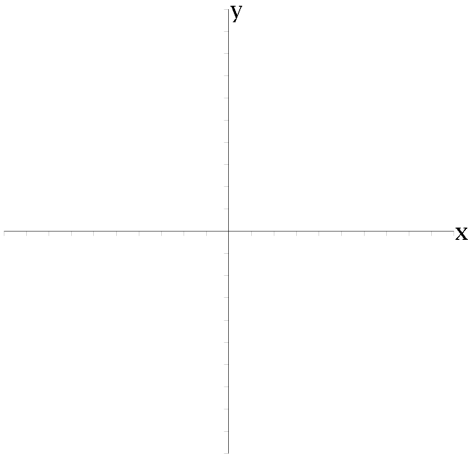
(b) What is the range of $y = \tan^{-1}(x^2)$?

(c) Determine the x-intercept(s) of the graph $y = e^{x^2-1} - 1$.

(d) Determine the y-intercept(s) of the graph $y = 2^{1/(x+1)} - 4$.

(e) Compute $\lim_{x\to\infty}(\ln(1+2x) - \ln(1+3x))$.

(f) Sketch $y = \ln(x^2 - 1)$.



3. (X points) Find the derivative of the given function, evaluated at the given point. Simplify completely (no inverse trig functions in your answer!).

(a) $y = \tan(\sin^{-1}(t)), t = 0.$

(b) $y = (\ln x)^{e^x}, x = e.$

4. (X points) Determine the correct partial fractions expansions. DO NOT DETERMINE THE CONSTANTS AND DO NOT INTEGRATE.

(a) To integrate

$$\int \frac{x}{x^2-7x+12}\,dx,$$

which partial fractions expansion should you use? DO NOT DETERMINE THE CONSTANTS AND DO NOT INTEGRATE.

(b) To integrate

$$\int \frac{x}{(x^2-1)^2(x^2-3x+12)^2}\,dx,$$

which partial fractions expansion should you use? DO NOT DETERMINE THE CONSTANTS AND DO NOT INTEGRATE.

5. (X points) Integrate by parts

$$\int_0^1 x \tan^{-1} x \, dx$$

6. (X points) Integrate using a trigonometric substitution. Simplify as much as you can.

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx$$

7. (X points) Integrate using whatever method you wish.

(a) $\displaystyle\int_0^{\pi/4} \cos^2\theta \, \sin^2\theta \, d\theta$

(b) $\displaystyle\int \sin(t)\ln(\cos^2(t))\, dt$

(c) $\int \frac{x}{\sqrt{x-x^2}}\,dx$

(d) $\int_2^3 \frac{x^2}{x^2-1}\,dx$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Co-function Identities

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \quad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right) \quad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) \quad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

Opposite Angle Identities

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

Sum and Difference Identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Power reduction formulae

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}, \quad \cos\theta \neq 1$$

Product-Sum Identities

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A + B) + \cos(A - B)\right]$$

Basic Integration Formulae

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq 1)$

2. $\int \frac{1}{x}\,dx = \ln|x| + C$

3. $\int e^x\,dx = e^x + C$

4. $\int a^x\,dx = \frac{a^x}{\ln a} + C$

5. $\int \sin x\,dx = -\cos x + C$

6. $\int \cos x\,dx = \sin x + C$

7. $\int \sec^2 x\,dx = \tan x + C$

8. $\int \csc^2 x\,dx = -\cot x + C$

9. $\int \sec x \tan x\,dx = \sec x + C$

10. $\int \csc x \cot x\,dx = -\csc x + C$

11. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

12. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$

13. $\int \tan x\,dx = \ln|\sec x| + C$

14. $\int \cot x\,dx = \ln|\sin x| + C$

15. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

16. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C, \;\; a > 0$