Induction proofs - practice!

1. Prove that $f(n) = 6n^2 + 2n + 15$ is odd for all $n \in \mathbb{Z}^+$.

2. Prove that $n! > 2^n$ for $n \geq 4$.

3. If n is a non-negative integer, show that $n^5 - n$ is divisible by 5.

4. For each $n \in \mathbb{N}$, it follows that $2^n \leq 2^{n+1} - 2^{n-1} - 1$. Show using induction.

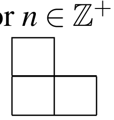
5. Prove that $(1+2+3+\cdots+n)^2 = 1^3 + 2^3 + 3^3 + \cdots + n^3$ for every $n \in \mathbb{Z}^+$.

6. Prove by induction that $1+2+3+4+...+n = n(n+1)/2$ for $n \in \mathbb{Z}^+$.

7. Prove by induction that $1+3+5+7+...+(2n-1) = n^2$ for $n \in \mathbb{Z}^+$.

8. Show by induction that $2^1 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$ for $n \in \mathbb{Z}^+$.

9. For n an integer greater or equal to 3, show that $n^{n+1} > (n+1)^n$.

10. Prove that for $n \in \mathbb{Z}^+$, a $2^n \times 2^n$ chessboard with any one square removed can be tiled by these 3-square L-tiles: 

11. Take any natural number n. If n is even, divide it by 2 to get $n/2$. If n is odd, multiply it by 3 and add 1 to obtain $3n+1$. Repeat the process indefinitely. Paul Erdös will owe you \$500 if you can show that, no matter what number n you start with, this process will eventually hit 1.

But be warned: link.