Homework 2

due Sept 2, 2015

In addition to

6.3. 1, 5, 7, 11, 17, 21, 25, 27, 29, 31, 33, 39, 47, 49, 51, 57, 59, 67

6.4. 7, 9, 17, 18, 27, 33, 42, 43, 45, 49, 53, 55, 57, 71, 73, 75, 77, 79, 81, 93

6.5. 3, 9, 12, 13, 17

complete the following problems.

1. Shannon Entropy In a recent application of information theory to the field of genomics, a function called the Shannon entropy, H, was considered. A given gene is represented as a binary device: it can be either "on" or "off" (i.e. being expressed or not). If x is the probability that the gene is "on" and y is the probability that it is "off", the Shannon entropy function for the gene is defined as

$$H = -x\ln(x) - y\ln(y)$$

(*Note*: the fact that x and y are probabilities means that they satisfy $0 < x \leq 1$, and $0 < y \leq 1$.) The gene can only be in one of these two states, so $x + y = 1$. Use these facts to show that the Shannon entropy for the gene is greatest when the two states are equally probable, i.e. for $x = y = 0.5$.

2. Uncontrolled growth In Michael Crichton's "The Andromeda Strain" (1969), there's a passage on uncontrolled growth:

The mathematics of uncontrolled growth are frightening. A single cell of the bacterium E. coli would, under ideal circumstances, divide every twenty minutes. That is not particularly disturbing until you think about it, but the fact is that bacteria multiply geometrically: one becomes two, two become four, four become eight, and so on. In this way it can be shown that in a single day, one cell of E. coli could produce a super-colony equal in size and weight to the entire planet Earth.

Did Crichton do his math right? Determine when the super-colony would match the mass of the earth. Use the mass of the earth as 5.972×10^{24} kg, the mass of a single E. coli cell is 1 ng, and the approximation $2^{10} \approx 10^3$.

Challenge problem. For $x > 0$, let $f(x) = x^x$. Find all values of x for which $f(x) = f'(x)$.

Coding preparation Read Section 2 (pages 3-7), trying the different plots as you go. Turn .pdf versions of the two plots described in the "Exercise" on page 7. *How to generate the .pdf to turn in:*e With the Figure window open with you plot, go to File –> Print, then under "Name" select "PDF print" and hit "OK." The software will take you through the steps and save your figure to your webfiles directory.

UPLOAD the two .pdf files to the ANGEL "Homework 2 plots" DROPBOX in Lessons–>Homework–>.

TO BE GRADED

6.3. 7, 11, 17, 21, 25, 29, 33, 39, 47, 51, 57, 59
6.4. 7, 18, 27, 33, 42, 43, 49, 53, 55, 77, 79, 81
6.5. 9, 13, 17

Uncontrolled growth problem

Challenge problem

Coding preparation problem

Note: EACH person must turn in their plots from the **Coding Preparation** problem individually (in ANGEL Dropbox).