TABLE 5.4 Parameter Values for the Transmission Dynamics of Community-Acquired and Hospital-Acquired Methicillin-Resistant *Staphylococcus aureus* Colonization (CA-MRSA and HA-MRSA)

Parameter	Symbol	Baseline Value
Total number of patients	**N**	**400**
Length of stay		
Susceptible	$1/\delta_S$	5 days
Colonized CA-MRSA	$1/\delta_C$	7 days
Colonized HA-MRSA	$1/\delta_H$	5 days
Transmission rate per susceptible patient to		
Colonized CA-MRSA per colonized CA-MRSA	β_C	0.45 per day
Colonized HA-MRSA per colonized HA-MRSA	β_H	0.4 per day
Decolonization rate per colonized patient per day per length of stay		
CA-MRSA	α_C	0.1 per day
HA-MRSA	α_H	0.1 per day

References

1. D'Agata, E. M. C., Webb, G. F., Pressley, J. 2010. Rapid emergence of co-colonization with community-acquired and hospital-acquired methicillin-resistant *Staphylococcus aureus* strains in the hospital setting. *Mathematical Modelling of Natural Phenomena* 5(3): 76–93.
2. Pressley, J., D'Agata, E. M. C., Webb, G. F. 2010. The effect of co-colonization with community-acquired and hospital-acquired methicillin-resistant *Staphylococcus aureus* strains on competitive exclusion. *Journal of Theoretical Biology* 265(3): 645–656.

C Things That Bob

Courtesy of Richard Bernatz, Department of Mathematics, Luther College

The motion of various-shaped objects that bob in a pool of water can be modeled by a second-order differential equation derived from Newton's second law of motion, $F = ma$. The forces acting on the object include the force due to gravity, a frictional force due to the motion of the object in the water, and a buoyant force based on **Archimedes' principle:** An object that is completely or partially submerged in a fluid is acted on by an upward (buoyant) force equal to the weight of the water it displaces.

(a) The first step is to write down the governing differential equation. The dependent variable is the depth z of the object's lowest point in the water. Take z to be negative downward so that $z = -1$ means 1 ft of the object has submerged. Let $V(z)$ be the submerged volume of the object, m be the mass of the object, ρ be the density of water (in pounds per cubic foot), g be the acceleration due to gravity, and γ_w be the coefficient of friction for water. Assuming that the frictional force is proportional to the vertical velocity of the object, write down the governing second-order ODE.

(b) For the time being, neglect the effect of friction and assume the object is a cube measuring L feet on a side. Write down the governing differential equation for this case. Next, designate $z = l$ to be the depth of submersion such that the buoyant force is equal and opposite the gravitational force. Introduce a new variable, ζ, that gives the displacement of the object from its equilibrium position l (that is, $z = \zeta + l$). You can now write the ODE in a more familiar form. [*Hint:* Recall the mass–spring system and the equilibrium case.] Now you should recognize the type of solution for this problem. What is the natural frequency?

(c) In this task you consider the effect of friction. The bobbing object is a cube, 1 ft on a side, that weighs 32 lb. Let $\gamma_w = 3$ lb-sec/ft, $\rho = 62.57$ lb/ft^3, and suppose the object is initially placed on the surface of the water. Solve the governing ODE by hand to find the general solution. Next, find the particular solution for the case in which the cube is initially placed on the surface of the water and is given no initial velocity. Provide a plot of the position of the object as a function of time t.

(d) In this step of the project, you develop a numerical solution to the same problem presented in part (c). The numerical solution will be useful (indeed necessary) for subsequent parts of the project. This case provides a trial to verify that your numerical solution is correct. Go back to the initial ODE you developed in part (a). Using parameter values given in part (c), solve the initial value problem for the cube starting on the surface with no initial velocity. To solve this problem numerically, you will have to write the second-order ODE as a system of two first-order ODEs, one for vertical position z and one for vertical velocity w. Plot your results for vertical position as a function of time t for the first 3 or 4 sec and compare with the analytical solution you found in part (c). Are they in close agreement? What might you have to do in order to compare these solutions? Provide a plot of both your analytical and numerical solutions on the same graph.

(e) Suppose a sphere of radius R is allowed to bob in the water. Derive the governing second-order equation for the sphere using Archimedes' principle and allowing for friction due to its motion in the water. Suppose a sphere weighs 32 lb, has a radius of $1/2$ ft, and $\gamma_w = 3.0$ lb-sec/ft. Determine the limiting value of the position of the sphere without solving the ODE. Next, solve the governing ODE for the velocity and position of the sphere as a function of time for a sphere placed on the surface of the water. You will need to write the governing second-order ODE as a system of two ODEs, one for velocity and one for position. What is the limiting position of the sphere for your solution? Does it agree with the equilibrium solution you found above? How does it compare with the equilibrium position of the cube? If it is different, explain why.

(f) Suppose the sphere in part (d) is a volleyball. Calculate the position of the sphere as a function of time t for the first 3 sec if the ball is submerged so that its lowest point is 5 ft under water. Will the ball leave the water? How high will it go? Next, calculate the ball's trajectory for initial depths lower than 5 ft and higher than 5 ft. Provide plots of velocity and position for each case and comment on what you see. Specifically, comment on the relationship between the initial depth of the ball and the maximum height the ball eventually attains.

You might consider taking a volleyball into a swimming pool to gather real data in order to verify and improve on your model. If you do so, report the data you found and explain how you used it for verification and improvement of your model.

D Hamiltonian Systems

The problems in this project explore the **Hamiltonian**[†] **formulation** of the laws of motion of a system and its phase plane implications. This formulation replaces Newton's second law $F = ma = my''$ and is based on three mathematical manipulations:

[†]*Historical Footnote:* Sir William Rowan Hamilton (1805–1865) was an Irish mathematical physicist. Besides his work in mechanics, he invented quaternions and discovered the anticommutative law for vector products.