MATHEMATICS 215 Midterm #2, Wednesday, March 25, 2009

Both questions are of equal value. Time Limit: 50 minutes
NO CALCULATORS OR OTHER ELECTRONIC DEVICES ARE PERMITTED.

1. Solve the initial value problem

$$\frac{dx}{dt} = 2x - y,$$
$$\frac{dy}{dt} = 4x - 3y + e^t$$

where $x(0) = 1,\ y(0) = 0.$

2. (a) Let $f(t) = \begin{cases} 1, & 0 \le t < h, \\ -1, & h \le t < 2h, \\ 0, & t \ge 2h. \end{cases}$

Find the Laplace transform of $f(t)$.

(b) Use the Laplace transform technique to solve the initial value problem

$$\frac{d^2y}{dt^2} + y = f(t), \quad t > 0$$

where $y(0) = 0,\ \left.\frac{dy}{dt}\right|_{t=0} = 0$ for $f(t)$ given in part (a).

(c) Sketch the solution obtained in part (b) for $h = 2\pi$.

TABLE OF INFORMATION

FUNCTION	**LAPLACE TRANSFORM**
$f(t)$	$F(s)$
$f'(t)$	$sF(s) - f(0)$
$u_a(t)$	$\frac{e^{-as}}{s}$
$u_a(t)\, f(t-a)$	$e^{-as}F(s)$
$\sin t$	$\frac{1}{s^2+1}$
$\cos t$	$\frac{s}{s^2+1}$
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
$tf(t)$	$-F'(s)$
$\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$