Math 255 Midterm Exam III

Nov 19, 2010

Duration: 50 minutes

Last Name: ________________ First Name: ____________ Student Number: ________

Do not open this test until instructed to do so! This exam should have 8 pages, including this cover sheet. It is a closed book exam; no textbooks, calculators, laptops, formula sheets or other aids are allowed. Turn off any cell phones, pagers, etc. that could make noise during the exam. You must remain in this room until you have finished the exam. **Circle your solutions! Reduce your answer as much as possible. Explain your work. Relax.** Use the extra pages if necessary.

Read these UBC rules governing examinations:

(i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem	**Out of**	**Score**
1	20	
2	20	
3	10	
Total	50	

Problem 1 (20 points)

Find the solution of the initial-value problem

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \vec{x} + \begin{bmatrix} -7 \\ 5 \end{bmatrix}, \qquad \vec{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

Problem 1 continued

Problem 2 (20 points)

A bottle of water, initially at room temperature (23°C), is put in a refrigerator, where the temperature is held at 3°C. After 6 hours it is removed from the refrigerator and returned to a room-temperature (23°C) environment. Assume the temperature of the water obeys Newton's law of cooling,

$$\frac{dy}{dt} = -k(y - T_a),$$

where $y(t)$ is the temperature of the water at time t ($y(0) = 23$°C), T_a is the ambient temperature, and k some constant of proportionality; take $k = 1$. In this case the ambient temperature in °C is given by

$$T_a = \begin{cases} 3, & 0 \le t < 6 \\ 23, & t \ge 6. \end{cases}$$

(a) Use Laplace transforms to find the temperature $y(t)$ at time t.

Problem 2 continued

(a) CONTINUED

Problem 2 continued

(b) Say you prefer to drink water that is warmer than the temperature in the fridge, at 13°C. When should you drink the water?

Problem 3 (10 points)

Consider the linear system of equations

$$\frac{dx}{dt} = x + \alpha y$$
$$\frac{dy}{dt} = y.$$

Place a restriction on α so that the general solution is of the form

$$\vec{x} = C_1 \vec{\xi} e^t + C_2 \left(\vec{\xi} t + \vec{\eta} \right) e^t$$

where $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ for some non-zero vectors $\vec{\xi}$ and $\vec{\eta}$. Justify your restriction.
NOTE: You DO NOT need to find the general solution.

Table of Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}1\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$1/s, \quad s > 0$
2. e^{at}	$1/(s-a), \quad s > a$
3. t^n, $n =$ positive integer	$n!/s^{n+1}, \quad s > 0$
4. t^p, $p > -1$	$\Gamma(p+1)/s^{n+1}$, $s > 0$
5. $\sin(at)$	$a/(s^2+a^2), \quad s > 0$
6. $\cos(at)$	$s/(s^2+a^2), \quad s > 0$
7. $\sinh(at)$	$a/(s^2-a^2), \quad s > \|a\|$
8. $\cosh(at)$	$s/(s^2-a^2), \quad s > \|a\|$
9. $e^{at}\sin(bt)$	$b/[(s-a)^2+b^2], \quad s > a$
10. $e^{at}\cos(bt)$	$(s-a)/[(s-a)^2+b^2], \quad s > a$
11. $t^n e^{at}$	$n!/(s-a)^{n+1}, \quad s > a$
12. $u_c(t)$	$e^{-cs}/s, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s),$
14. $e^{ct}f(t)$	$F(s-c),$
15. $f(ct)$	$F(s/c)/c, \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - ... - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$