Math 255 Midterm Exam III

Nov 19, 2010

Duration: 50 minutes

Last Name: ________________ First Name: ____________ Student Number: _______

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Problem	**Out of**	**Score**
1	20	
2	20	
3	10	
Total	50	

Problem 1 (20 points)

Find the solution of the initial-value problem

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \vec{x} + \begin{bmatrix} -7 \\ 5 \end{bmatrix}, \qquad \vec{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

SOLUTION:

First we find the homogeneous solution:

$$\frac{d\vec{x}_h}{dt} = A\vec{x}_h \text{ where } A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}.$$

We need the eigenvectors and eigenvalues of A. The characteristic equation is

$$\det(A - rI) = 0 \Rightarrow \det \begin{bmatrix} 2-r & 3 \\ -1 & -2-r \end{bmatrix} = 0 \Rightarrow r^2 - 1 = 0.$$

Thus our eigenvalues are $r_1 = 1$ and $r_2 = -1$. For each we need corresponding the eigenvector. The eigenvector $\vec{\xi}_1$ corresponding to the eigenvector $r_1 = 1$ satisfies:

$$(A - I)\vec{\xi} = 0 \text{ or } \begin{bmatrix} 1 & 3 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Then ξ_1 and ξ_2 satisfy the scalar equation $\xi_1 + 3\xi_2 = 0$; choose $\vec{\xi} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. The eigenvector $\vec{\xi}_2$ corresponding to the eigenvector $r_2 = -1$ satisfies:

$$(A + I)\vec{\xi} = 0 \text{ or } \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

This time ξ_1 and ξ_2 satisfy the scalar equation $\xi_1 + \xi_2 = 0$; choose $\vec{\xi} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Together, we find that the fundamental solution set is

$$\left\{ \begin{bmatrix} 3 \\ -1 \end{bmatrix} e^t, \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} \right\}.$$

We can use this to write down a homogeneous solution, but then we also need a particular solution before solving for the constants. We use three different methods below.

Method I - Undetermined Coefficients

From the fundamental solution set we know that the homogeneous solution is

$$\vec{x}_h = C_1 \begin{bmatrix} 3 \\ -1 \end{bmatrix} e^t + C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}.$$

Problem 1 continued

To find the particular solution, use the trial guess $\vec{x}_p(t) = \vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$. Then

$$\frac{d\vec{x}_p}{dt} = A\vec{x}_p + \begin{bmatrix} -7 \\ 5 \end{bmatrix} \Rightarrow \vec{0} = A\vec{a} + \begin{bmatrix} -7 \\ 5 \end{bmatrix} \Rightarrow \vec{a} = A^{-1}\begin{bmatrix} -7 \\ 5 \end{bmatrix}.$$

Now since $A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ (yes - it is the same!). Computing $\vec{a}$, $\vec{a} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$. Therefore the general solution is

$$\begin{aligned} \vec{x}(t) &= \vec{x}_h(t) + \vec{x}_p(t) \\ &= C_1 \begin{bmatrix} 3 \\ -1 \end{bmatrix} e^t + C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}. \end{aligned}$$

All that's left is to solve for constants: $\vec{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. Then

$$\begin{aligned} \begin{bmatrix} 2 \\ 0 \end{bmatrix} &= C_1 \begin{bmatrix} 3 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 3C_1 + C_2 - 1 \\ -C_1 - C_2 + 3 \end{bmatrix}. \end{aligned}$$

Solving simultaneously find $C_1 = 0$, $C_2 = 3$. The solution to the given IVP is

$$\vec{x}(t) = 3\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3e^{-t} - 1 \\ -3e^{-t} + 3 \end{bmatrix}.$$

Method II - Variation of Parameters

From the fundamental solution set we can write down the fundamental matrix

$$\Psi(t) = \begin{bmatrix} 3e^t & e^{-t} \\ -e^t & -e^{-t} \end{bmatrix}.$$

Note that $\Psi'(t) = A\Psi(t)$. Let $\vec{x}(t) = \Psi(t)\vec{u}(t)$. Then

$$\begin{aligned} \vec{x}'(t) &= A\vec{x}(t) + \begin{bmatrix} -7 \\ 5 \end{bmatrix} \\ \Psi'(t)\vec{u}(t) + \Psi(t)\vec{u}'(t) &= A\Psi(t)\vec{u}(t) + \begin{bmatrix} -7 \\ 5 \end{bmatrix} \\ A\Psi(t)\vec{u}(t) + \Psi(t)\vec{u}'(t) &= A\Psi(t)\vec{u}(t) + \begin{bmatrix} -7 \\ 5 \end{bmatrix} \\ \Psi(t)\vec{u}'(t) &= \begin{bmatrix} -7 \\ 5 \end{bmatrix} \\ \vec{u}'(t) &= \Psi(t)^{-1}\begin{bmatrix} -7 \\ 5 \end{bmatrix}. \end{aligned}$$

Problem 1 continued

Since $\Psi(t) = \begin{bmatrix} 3e^t & e^{-t} \\ -e^t & -e^{-t} \end{bmatrix}$, $\Psi(t)^{-1} = \begin{bmatrix} e^{-t}/2 & e^{-t}/2 \\ -e^t/2 & -3e^t/2 \end{bmatrix}$ and assuming $\vec{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$,

$$\begin{bmatrix} u_1'(t) \\ u_2'(t) \end{bmatrix} = \begin{bmatrix} -e^{-t} \\ -4e^t \end{bmatrix}.$$

Integrating, we find

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} e^{-t} + C_1 \\ -4e^t + C_2 \end{bmatrix}.$$

The general solution is therefore

$$\vec{x}(t) = \Psi(t)\vec{u}(t) = \begin{bmatrix} 3e^t & e^{-t} \\ -e^t & -e^{-t} \end{bmatrix} \begin{bmatrix} e^{-t} + C_1 \\ -4e^t + C_2 \end{bmatrix} = \begin{bmatrix} -1 + 3C_1e^t + C_2e^{-t} \\ 3 - C_1e^t - C_2e^{-t} \end{bmatrix}.$$

All that's left is to solve for constants: $\vec{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. Then

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 + 3C_1 + C_2 \\ 3 - C_1 - C_2 \end{bmatrix}.$$

Solving simultaneously find $C_1 = 0$, $C_2 = 3$. The solution to the given IVP is

$$\vec{x}(t) = \begin{bmatrix} 3e^{-t} - 1 \\ -3e^{-t} + 3 \end{bmatrix}.$$

Method III - Diagonalization

In finding the homogeneous solution above we found the eigenvalues and eigenvectors of A. Then $A = TDT^{-1}$ where $T = \begin{bmatrix} 3 & 1 \\ -1 & -1 \end{bmatrix}$ (each column is an eigenvector) and $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (diagonal matrix of eigenvalues). Then

$$\begin{aligned} \frac{d\vec{x}}{dt} &= TDT^{-1}\vec{x} + \begin{bmatrix} -7 \\ 5 \end{bmatrix} \\ \frac{d}{dt}(T^{-1}\vec{x}) &= DT^{-1}\vec{x} + T^{-1}\begin{bmatrix} -7 \\ 5 \end{bmatrix} \end{aligned}$$

Let $\vec{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = T^{-1}\vec{x}(t)$ and noting that

$$T^{-1}\begin{bmatrix} -7 \\ 5 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & -3/2 \end{bmatrix}\begin{bmatrix} -7 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix},$$

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} -1 \\ -4 \end{bmatrix}.$$

Problem 1 continued

We now have two, decoupled, first-order ODEs for y_1 and y_2 (this is beauty of the diagonalization method).

$$\frac{dy_1}{dt} = y_1 - 1 \Rightarrow y_1 = 1 + C_1 e^t$$

$$\frac{dy_2}{dt} = -y_2 - 4 \Rightarrow y_2 = -4 + C_2 e^{-t}.$$

Then since $\vec{x} = T\vec{y}$ the general solution is:

$$\begin{aligned} \vec{x} &= \begin{bmatrix} 3 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 + C_1 e^t \\ -4 + C_2 e^{-t} \end{bmatrix} \\ &= \begin{bmatrix} -1 + 3C_1 e^t + C_2 e^{-t} \\ 3 - C_1 e^t - C_2 e^{-t} \end{bmatrix}. \end{aligned}$$

All that's left is to solve for constants: $\vec{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. Then

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 + 3C_1 + C_2 \\ 3 - C_1 - C_2 \end{bmatrix}.$$

Solving simultaneously find $C_1 = 0$, $C_2 = 3$. The solution to the given IVP is

$$\vec{x}(t) = \begin{bmatrix} 3e^{-t} - 1 \\ -3e^{-t} + 3 \end{bmatrix}.$$

Answer:

No matter what method one uses, the solution to the given initial value problem is:

$$\vec{x}(t) = 3 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + \begin{bmatrix} -1 \\ 3 \end{bmatrix} \quad \text{or} \quad \vec{x}(t) = \begin{bmatrix} 3e^{-t} - 1 \\ -3e^{-t} + 3 \end{bmatrix}.$$

Problem 2 (20 points)

A bottle of water, initially at room temperature (23°C), is put in a refrigerator, where the temperature is held at 3°C. After 6 hours it is removed from the refrigerator and returned to a room-temperature (23°C) environment. Assume the temperature of the water obeys Newton's law of cooling,

$$\frac{dy}{dt} = -k(y - T_a),$$

where $y(t)$ is the temperature of the water at time t ($y(0) = 23$°C), T_a is the ambient temperature, and k some constant of proportionality; take $k = 1$. In this case the ambient temperature in °C is given by

$$T_a = \begin{cases} 3, & 0 \le t < 6 \\ 23, & t \ge 6. \end{cases}$$

(a) Use Laplace transforms to find the temperature $y(t)$ at time t.

SOLUTION:

To be perfectly clear, let's re-write this as an initial value problem

$$\frac{dy}{dt} + y = \begin{cases} 3, & 0 \le t < 6 \\ 23, & t \ge 6 \end{cases}, \quad y(0) = 23.$$

Since we want to solve via Laplace transforms and would like to use tables, we write the ambient temperature (right hand side) in terms of step functions:

$$\begin{cases} 3, & 0 \le t < 6 \\ 23, & t \ge 6 \end{cases} = 3 + 20u_6(t).$$

Then we take Laplace transforms of the equation. Let $\mathcal{L}\{y(t)\} = Y(s)$. Then

$$\mathcal{L}\left\{\frac{dy}{dt} + y\right\} = \mathcal{L}\left\{3 + 20u_6(t)\right\}$$
$$\mathcal{L}\left\{\frac{dy}{dt}\right\} + \mathcal{L}\left\{y\right\} = 3\mathcal{L}\left\{1\right\} + 20\mathcal{L}\left\{u_6(t)\right\}.$$

Transforming each term individually,

$$\mathcal{L}\left\{\frac{dy}{dt}\right\} = sY(s) - y(0) = sY(s) - 23,$$

$$\mathcal{L}\{1\} = \frac{1}{s},$$

$$\mathcal{L}\{u_6(t)\} = \frac{e^{-6s}}{s}.$$

Therefore our transformed IVP is

$$(sY(s) - 23) + Y(s) = 3\left(\frac{1}{s}\right) + 20\left(\frac{e^{-6s}}{s}\right).$$

Problem 2 continued
(a) CONTINUED

Solving for $Y(s)$,

$$Y(s) = 3\left(\frac{1}{s(s+1)}\right) + 20\left(\frac{e^{-6s}}{s(s+1)}\right) + \frac{23}{(s+1)}.$$

Now to recover the solution $y(t)$ we have to take the inverse transform - $y(t) = \mathcal{L}^{-1}\left\{Y(s)\right\}$. Then

$$\begin{aligned} y(t) &= \mathcal{L}^{-1}\left\{3\left(\frac{1}{s(s+1)}\right) + 20\left(\frac{e^{-6s}}{s(s+1)}\right) + \frac{23}{(s+1)}\right\} \\ &= 3\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} + 20\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s(s+1)}\right\} + 23\mathcal{L}^{-1}\left\{\frac{1}{(s+1)}\right\}. \end{aligned}$$

Approach I - Using partial fractions expansion:
Note that

$$\frac{1}{s(s+1)} = \left(\frac{1}{s}\right) - \left(\frac{1}{s+1}\right).$$

Then

$$\begin{aligned} y(t) &= 3\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} + 20\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s(s+1)}\right\} + 23\mathcal{L}^{-1}\left\{\frac{1}{(s+1)}\right\} \\ &= 3\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 20\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s}\right\} - 20\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s+1}\right\} + 23\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} \\ &= 3\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 20\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 20\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s}\right\} - 20\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s+1}\right\}. \end{aligned}$$

From tables,

$$\begin{aligned} \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} &= 1, \\ \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} &= e^{-t}, \\ \mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s}\right\} &= u_6(t), \\ \mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s+1}\right\} &= e^{-(t-6)}u_6(t). \end{aligned}$$

In computing the last two inverses, we used the rule $\mathcal{L}^{-1}\left\{e^{-cs}F(s)\right\} = f(t-c)u_c(t)$ where $f(t) = \mathcal{L}^{-1}\left\{F(s)\right\}$. All together,

$$\begin{aligned} y(t) &= 3 + 20e^{-t} + 20u_6(t) - 20e^{-(t-6)}u_6(t) \\ y(t) &= 3 + 20e^{-t} + 20(1 - e^{-(t-3)})u_6(t). \end{aligned}$$

Problem 2 continued
(a) CONTINUED

Approach II - Using convolution integrals:
If we were using this approach there would be no need to compute the laplace transforms on the right hand side. Instead we would have

$$Y(s) = \frac{1}{s+1}\mathcal{L}\left\{3 + 20u_6(t)\right\} + \frac{23}{s+1}$$

and since, from tables, $\mathcal{L}\{e^{-t}\} = 1/(s+1)$,

$$Y(s) = \mathcal{L}\left\{e^{-t}\right\}\mathcal{L}\left\{3 + 20u_6(t)\right\} + 23\mathcal{L}\left\{e^{-t}\right\}.$$

Let $f(t) = e^{-t}$ with $F(s) = \mathcal{L}\left\{f(t)\right\} = \mathcal{L}\left\{e^{-t}\right\}$, and $g(t) = 3 + 20u_6(t)$ with $G(s) = \mathcal{L}\{g(t)\} = \mathcal{L}\left\{3 + 20u_6(t)\right\}$. Taking the inverse transform, with $y(t) = \mathcal{L}^{-1}\{Y(s)\}$,

$$y(t) = \mathcal{L}^{-1}\left\{F(s)G(s)\right\} + 23e^{-t}.$$

Using convolution integrals, with $F(s) = \mathcal{L}\left\{f(t)\right\}$ and $G(s) = \mathcal{L}\{g(t)\}$,

$$\begin{aligned}
\mathcal{L}^{-1}\left\{F(s)G(s)\right\} &= \int_0^t f(t-\tau)g(\tau)d\tau \\
&= \int_0^t e^{-(t-\tau)}\left(3 + 20u_6(\tau)\right)d\tau \\
&= 3e^{-t}\int_0^t e^{\tau}\,d\tau + 20e^{-t}u_6(t)\int_6^t e^{\tau}\,d\tau \\
&= 3e^{-t}\left[e^{\tau}\right]_0^t + 20e^{-t}u_6(t)\left[e^{\tau}\right]_6^t \\
&= 3 - 3e^{-t} + 20\left(1 - e^{-(t-6)}\right)u_6(t).
\end{aligned}$$

Then from above,

$$\begin{aligned}
y(t) &= \left(3 - 3e^{-t} + 20\left(1 - e^{-(t-6)}\right)u_6(t)\right) + 23e^{-t} \\
&= 3 + 20e^{-t} + 20\left(1 - e^{-(t-6)}\right)u_6(t).
\end{aligned}$$

Either way the final answer is:

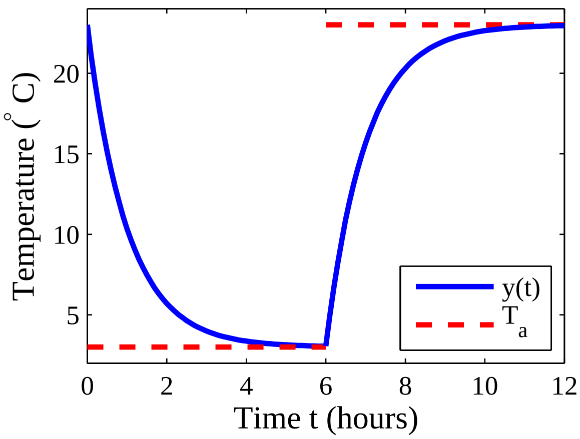
$$\mathbf{y(t) = 3 + 20e^{-t} + 20\left(1 - e^{-(t-6)}\right)u_6(t)}.$$

Problem 2 continued

(b) Say you prefer to drink water that is warmer than the temperature in the fridge, at 13°C. When should you drink the water?

SOLUTION:

The temperature $y(t)$ is 13°C at two distinct times, one for $0 \leq t < 6$ and one for $t \geq 6$. This is especially clear if you plot the solution (which you were not required to do):



(Notice that the temperature exponentially approaches the ambient temperature, whatever it should be at the time.)

Let t_1 be the time between $t = 0$ and $t = 6$. Then t_1 satisfies:

$$\begin{aligned} 13 &= 3 + 20e^{-t_1} \\ t_1 &= \ln(2). \end{aligned}$$

Let t_2 be the time for $t \geq 6$. Then t_2 satisfie

$$\begin{aligned} 13 &= 23 - 20e^{-t_2}(e^6 - 1) \\ t_2 &= \ln(2(e^6 - 1)). \end{aligned}$$

Therefore the temperature of the water is $13°C$ after $t_1 = \ln(2)$ and $t_2 = \ln(2(e^6 - 1))$ hours.

Problem 3 (10 points)

Consider the linear system of equations

$$\frac{dx}{dt} = x + \alpha y$$
$$\frac{dy}{dt} = y.$$

Place a restriction on α so that the general solution is of the form

$$\vec{x} = C_1 \vec{\xi} e^t + C_2 \left(\vec{\xi} t + \vec{\eta} \right) e^t$$

where $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ for some non-zero vectors $\vec{\xi}$ and $\vec{\eta}$. Justify your restriction.

NOTE: You DO NOT need to solve this problem. Do not compute $\vec{\xi}$ or $\vec{\eta}$.

SOLUTION:

Writing this as a linear system:

$$\vec{x}'(t) = A\vec{x}(t) \ \text{ where } \ \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}; \text{and } A = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}.$$

We observe from the general solution that we want α such that A has a repeated eigenvalue with one linearly independent eigenvector.

Eigenvalues: The eigenvalues of A are the roots r of $\det(A - rI) = 0$:

$$det \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} = 0$$

$$\text{or } (1 - r)^2 = 0.$$

Then $r = 1$ is a repeated root no matter what α is.

Eigenvectors: The eigenvector(s) $\vec{\xi}$ of A corresponding to the repeating eigenvalue $r = 1$ satisfies $(A - I)\vec{\xi} = 0$ or, letting $\vec{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$,

$$\begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = 0.$$

Observe that there is only one linearly independent eigenvector, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, *unless* $\alpha = 0$, in which case there are two.

Answer: Therefore if $\alpha \neq 0$ the general solution is of the form given.

Table of Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$1/s, \quad s > 0$
2. e^{at}	$1/(s-a), \quad s > a$
3. t^n, $n =$ positive integer	$n!/s^{n+1}, \quad s > 0$
4. t^p, $p > -1$	$\Gamma(p+1)/s^{n+1}$, $s > 0$
5. $\sin(at)$	$a/(s^2 + a^2), \quad s > 0$
6. $\cos(at)$	$s/(s^2 + a^2), \quad s > 0$
7. $\sinh(at)$	$a/(s^2 - a^2), \quad s > \vert a \vert$
8. $\cosh(at)$	$s/(s^2 - a^2), \quad s > \vert a \vert$
9. $e^{at}\sin(bt)$	$b/[(s-a)^2 + b^2], \quad s > a$
10. $e^{at}\cos(bt)$	$(s-a)/[(s-a)^2 + b^2], \quad s > a$
11. $t^n e^{at}$	$n!/(s-a)^{n+1}, \quad s > a$
12. $u_c(t)$	$e^{-cs}/s, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s),$
14. $e^{ct}f(t)$	$F(s-c),$
15. $f(ct)$	$F(s/c)/c, \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$