
MATH 255 – Midterm I

October 16, 2006

Your full name: _____

ID Number: _____

Scores:

Problem 1 _____ (5 points)

Problem 2 _____ (5 points)

Problem 3 _____ (5 points)

Problem 4 _____ (5 points)

Problem 5 _____ (5 points)

TOTAL: _____ (25 points)

One 8.5×11 sheet of notes allowed.

Show all your work and make your reasoning clear.

Problem 1 (5 points)

Find the solution of

$$y' = x + 2xy, \quad y(0) = 1.$$

Problem 2 (5 points)

Find the general solution of

$$y'' - 5y' + 6y = 4xe^x$$

Problem 3 (5 points)

Find the general solution of

$$y' = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

Hint: Use the substitution $z = \frac{y}{x}$.

Problem 4 (5 points)

Consider a cylindrical tank of constant cross section A . Water is pumped into the tank at a constant rate k and leaks out through a small hole of area a in the bottom. Let $h = h(t)$ be the depth of water in the tank at time t . Due to Torricelli's principle, it satisfies the differential equation

$$Ah' = (k - \alpha a\sqrt{2gh}),$$

where g is the acceleration due to gravity, and α is a contraction coefficient with $0.5 \leq \alpha \leq 1$. Determine the equilibrium depth h_e of water and discuss the asymptotic stability of the equilibrium solution.

Problem 5 (5 points)

Consider

$$y'' - \frac{1}{2x}y' + \frac{1}{2x^2}y = 0, \quad x \geq 1.$$

Show that the functions $y_1(x) = x$ and $y_2(x) = \sqrt{x}$ solve the equation and form a fundamental set of solutions.

