

### Forced Oscillations Example

Consider the forced LCR circuit, which can be described by:

$$Q''(t) + RQ'(t) + \frac{1}{C}Q = F \cos(\omega t), \quad Q(0) = 0, \quad Q'(0) = 1,$$

where  $Q(t)$  is the charge at time  $t$ ,  $R$  the resistance,  $C$  the capacitance, and the inductance  $L = 1$ . The forcing has amplitude  $F$  and frequency  $\omega$ .

On the following page you will find plots of different solutions  $Q(t)$  for various values of  $R$ ,  $C$ ,  $F$ , and  $\omega$ , corresponding in no particular order to:

- A  $R = 1, C = 1/100, F = 1, \omega = 9 \Rightarrow$   
 $Q''(t) + Q'(t) + 100Q = \cos(9t), \quad Q(0) = 0, \quad Q'(0) = 1.$
- B  $R = 0, C = 1/16, F = 1, \omega = 4 \Rightarrow$   
 $Q''(t) + 16Q = \cos(4t), \quad Q(0) = 0, \quad Q'(0) = 1.$
- C  $R = 0, C = 1/16, F = 0, \omega$  not relevant  $\Rightarrow$   
 $Q''(t) + 16Q = 0, \quad Q(0) = 0, \quad Q'(0) = 1.$
- D  $R = 1, C = 1/100, F = 1, \omega = 10 \Rightarrow$   
 $Q''(t) + Q'(t) + 100Q = \cos(10t), \quad Q(0) = 0, \quad Q'(0) = 1.$
- E  $R = 1, C = 1/100, F = 0, \omega$  not relevant  $\Rightarrow$   
 $Q''(t) + Q'(t) + 100Q = 0, \quad Q(0) = 0, \quad Q'(0) = 1.$
- F  $R = 0, C = 1/100, F = 0, \omega$  not relevant  $\Rightarrow$   
 $Q''(t) + 100Q = 0, \quad Q(0) = 0, \quad Q'(0) = 1.$
- G  $R = 8.1, C = 1/100, F = 0, \omega$  not relevant  $\Rightarrow$   
 $Q''(t) + 8.1Q'(t) + 16Q = 0, \quad Q(0) = 0, \quad Q'(0) = 1.$
- H  $R = 0, C = 1/100, F = 1, \omega = 9 \Rightarrow$   
 $Q''(t) + 100Q = \cos(9t), \quad Q(0) = 0, \quad Q'(0) = 1.$

Match the differential equation with its solution's plot by writing the appropriate letter in the box to the *left* of the plot, on the following page.

**Hints:**

1. You don't need to solve each problem completely.
2. Look at the plots carefully and don't forget to check the axis scales.

