Math 257/316 Assignment 3
Due Wednesday October 7th in class

Problem 1: In the following differential equations, verify that the given point is a regular singular point, then find the indicial equation and the exponents for the specified singularity.
(a) $x^2y'' - 2xy' - 10y = 0$ at $x = 0$
(b) $(x^2 - x - 2)^2y'' + (x^2 - 4)y' - 6xy = 0$ at $x = 2$
(c) $x^3y'' + x\sin(x)y' - \tan(x)y = 0$ at $x = 0$

Problem 2: Use the method of Frobenius to find at least the first three nonzero terms in the series expansion about $x = 0$ for one of the two linearly independent solutions to the following differential equations for $x > 0$.
(a) $9x^2y'' + 9x^2y' + 2y = 0$
(b) $x^2y'' - xy' + \left(x^2 + \frac{3}{4}\right)y = 0$

Problem 3: For the following equation, verify that $x = 0$ is a regular singular point, find the indicial equation, the exponents at the singularity, the recurrence relation, and the first three nonzero terms for the solution corresponding to the larger root of the indicial equation.

$$x^2y'' - 2xy' + (x + 2)y = 0, \quad x > 0$$

Problem 4: Verify that $x = 0$ is a regular singular point of the following equation. Find the first three nonzero terms in each of two linearly independent solutions about $x = 0$.

$$2x^2y'' - xy' + (1 + x^2)y = 0.$$

Problem 5: In some applications, it is desirable to have an expansion about the point at infinity. To obtain such an expansion, we use the change of variables $z = 1/x$ and expand about $z = 0$.
Consider the following differential equation:

$$x^3y'' + \frac{3}{2}x^2y' - y = 0$$

(a) Show that infinity is a regular singular point of the given differential equation by showing that $z = 0$ is a regular singular point for the transformed equation in terms of z.
(b) Find the first three nonzero terms in the series expansion about infinity of one of the solutions of the original differential equation in terms of x.