

## MATH 257/316 Assignment 2

Due: Wed, Sept 30 in class

### Problem 1:

Consider the IVP (initial value problem):

$$(x - 1)y'' - (x - 3)y' - y = 0, \quad y(0) = 3, \quad y'(0) = 3.$$

Determine a power series solution centred about  $x_0 = 0$ .

Rewrite your answer in terms of elementary functions.

### Problem 2:

By using a power series representation centred about  $x_0 = 0$  determine the general solution of

$$(x^2 + 1)y'' + \frac{7}{2}xy' + y = 0.$$

### Problem 3:

Substitute the power series  $\sum_{n=0}^{\infty} a_n x^n$  into

$$y'' + x^2 y' + y = 0,$$

and determine a recursion relation describing the  $a_n$ 's.

Compute  $a_2$ ,  $a_3$ ,  $a_4$  and  $a_5$  in terms of  $a_0$  and  $a_1$ .

### Problem 4:

Find all singular points of the following ODEs and determine whether each one is regular or irregular.

- a)  $x^2 y'' + 3y' + 2xy = 0$
- b)  $(x^2 - 1)^2 y'' + (x + 1)y' + (x + 2)y = 0$
- c)  $\sin^2(x)y'' + \sin(\frac{x}{2})y' + 2y = 0$

### Problem 5:

Determine a lower bound for the radius of convergence of series solutions for the given ODE about the corresponding point  $x_0$ . (You should not try to compute the series solution itself.)

- a)  $(x^2 - 3)y'' + y' + \tan(x)y = 0$ ,  $x_0 = 0$
- b)  $(4x^2 + 9)y'' + x^3 y' + y = 0$ ,  $x_0 = 2$