

## MATH 257/316 Assignment 4

Due: Fri, Oct 16 in class

### Problem 1:

Find all eigenvalues and eigenfunctions of

$$y'' + \lambda y = 0 \quad (0 < x < 5), \quad \text{with } y(0) = y'(5) = 0.$$

### Problem 2:

Use separation of variables to determine the solution of

$$\begin{aligned} u_t &= \alpha^2 u_{xx} \quad (0 < x < 1) \\ \text{BC : } u'(0, t) &= u'(1, t) = 0 \\ \text{IC : } u(x, 0) &= f(x) \end{aligned}$$

### Problem 3:

Determine the minimal (fundamental) period of

- a)  $\sec(3x)$
- b)  $\sin\left(\frac{x}{4}\right) + \sin\left(\frac{x}{7}\right)$
- c)  $\frac{1}{3 - \cos(x)}$
- d)  $\ln(2 + \sin^2(x))$

### Problem 4:

Express

$$f(x) = \begin{cases} 4 - 5x, & 0 < x < \frac{1}{2} \\ 5 - 5x, & \frac{1}{2} \leq x < 1 \end{cases}$$

as a Fourier sine series over the interval  $[0, 1]$ .

### Problem 5:

Sketch the odd, even and full periodic extensions on  $[-3L, 3L]$  of

- a)  $e^x$ , with  $L = 1$
- b)  $4 - x^2$ , with  $L = 2$
- c)  $g(x) = \begin{cases} 1 + x, & x < 0 \\ x/2, & x \geq 0 \end{cases}$ , with  $L = 1$