Math 257/316 Assignment 5

Due Friday October 30th in class

Problem 1: Sketch the odd, even, and full periodic extensions on $[3L, 3L]$ of

(a) e^x, with $L = 1$

(b) $4 - x^2$, with $L = 2$

(c) $g(x) = \begin{cases} 1 + x, & x < 0 \\ x/2, & x \geq 0 \end{cases}$, with $L = 1$

Problem 2: Chemical diffusion through a thin layer is governed by the equation

$$\frac{\partial C}{\partial t} = k\frac{\partial^2 C}{\partial x^2} - LC$$

where $C(x,t)$ is the concentration in moles/cm^3, the diffusivity k is a positive constant with units cm^2/sec, and $L > 0$ is a consumption rate with units sec^{-1}. Assume boundary conditions are

$$C(0,t) = C(a,t) = 0, \; t > 0,$$

and the initial concentration is given by

$$C(x,0) = f(x), \; 0 < x < a.$$

(a) Use the method of separation of variables to solve for the concentration $C(x,t)$.

(b) What happens to the concentration as $t \to \infty$?

(c) What is the concentration $C(x,t)$ if the initial condition is $C(x,0) = \cos(\pi x/a)$?

Hint: It may be useful to know that

$$\int_0^a \sin(n\pi x/a)\cos(\pi x/a)\,dx = \begin{cases} 0, & \text{if } n \text{ is odd} \\ \frac{2an}{\pi(n^2-1)}, & \text{if } n \text{ is even} \end{cases}$$

Problem 3: Find the Fourier Sine series of period 2π of the following function. Sketch the graph of the function to which the series converges (sketch at least three periods).

$$f(x) = \begin{cases} 1, & 0 \leq x \leq \pi/2 \\ 0, & \pi/2 < x \leq \pi \end{cases}$$