Math 257/316 Assignment 1
Due Wednesday September 23rd in class

Problem 1: A similarity solution: Let $s = xt^{-\frac{1}{2}}$ and look for a solution to the heat equation $u_t = u_{xx}$ which is of the form

$$u(x,t) = t^{-\frac{1}{2}} f(s) \tag{1}$$

and which satisfies the condition $\int_{-\infty}^{\infty} u(x,t)dx = 1$. (Hint: Plug (1) into the heat equation and gather the f and f' terms in the resulting ODE to form a product. It may be useful to use the fact that $\int_{-\infty}^{\infty} e^{-x^2}dx = \pi^{\frac{1}{2}}$).

Problem 2: If f and g are any twice differentiable functions show that $u(x,t) = f(x+ct) + g(x-ct)$ is a solution of the wave equation $u_{tt} = c^2 u_{xx}$.

Problem 3: Find the general solution of the following equations:

a. $2y'' - 5y' - 3y = 0$

b. $y^{(4)} + 2y'' + y = 0$

c $y'' - 5y' + 4y = 8e^x$
d.$x^2y'' + -2xy' - 4y = 0$
e.$4x^2y'' + 8xy' + y = 0$
f. $x^2y'' + 3xy' + 3y = 0$, $y(1) = 1$, $y'(1) = -5$
Problem 4: Consider the following first order linear ODE:

$$(1-x)y' - y = 0 \tag{2}$$

1. Solve this differential equation using an integrating factor.
2. Expand the above solution in a Taylor series about the point $x_0 = 0$. For what values of x does this series fail to converge?
3. Now assume a power series solution of the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Substitute this series into the differential equation (2) and obtain a recursion for the coefficients a_n. Use this recursion to determine the series representation of solution. Compare this result to the series obtained in part 2 above. Is there any relationship between the points of divergence of the series and the coefficient $(1-x)$ of the derivative in (2)?