HOMEWORK 1: MATH 265 Due in class on September 17th

<u>Problem 1:</u> In each case, solve for $y(t)$:

- a. $y' + 2y = 0$ with $y(0) = 1$.
- b. $y' + 2y = 3e^t$ with $y(0) = 0$.
- c. $y' + 2y = te^{-2t}$ with $y(1) = 0$.
- d. $ty' + 2y = \cos t$ with $y(\pi) = 0$.
- e. $t^3y' + 4t^2y = e^t$ with $y(1) = 0$.

<u>Problem 2:</u> A radioactive substance emits radiation and changes to a new substance. The rate of decay of the substance is proportional to the quantity of the original substance remaining. The half-life of a radioactive substance is the amount of time it takes for half the substance to decay radioactively. If the half-life of radium is 1760 years, how much radium is left from an initial gram of radium after 100 years?

<u>Problem 3:</u> In a murder investigation a corpse was found by Inspector Clouseau at exactly 8 : 00 p. m. Being alert, he measures the temperature of the body and finds it to be 70°F (Fahrenheit). Two hours later, Inspector Clouseau again measures the temperature of the corpse and finds it to be 60° F. If the room temperature is 50° F, and we assume that Newton's law of cooling applies, when did the murder occur? (Assume that the temperature of the body at the time of the murder was 98.6°).
Newton's law of cooling states that the temperature T of an object in a room at temperature T_0 follows the equation

$$\frac{dT}{dt} = k(T_0 - T).$$

<u>Problem 4:</u> In the previous problem, Inspector Clouseau concluded that the time of the murder was 2.6 hours before he took the first temperature reading, or at 5 : 23 p. m. However, someone points out that Clouseau's analysis is faulty because the room temperature in which the corpse was found was not constant but instead decreased exponentially according to the law $50e^{-.05t}$, where t is the time (in hours) starting from 8 : 00 p.m. Assume that the room temperature obeys this law.

- (i) What is the differential equation that Inspector Clouseau must solve now?
- (ii) What is the temperature of the body at any time t?
- (iii) When was the time of the murder?

Problem 5: A spherical raindrop evaporates at a rate proportional to its surface area. If the radius of the raindrop is initially 3 mm and after 1/2 of an hour it has been reduced to 2 mm, find the time at which the raindrop has completely evaporated.

Problem 6: A ball of mass m falls from rest from a height h towards the ground. We assume that the ball is acted upon by a constant gravitational force and by an opposing frictional force, which is proportional to the velocity. Thus, the velocity $v = v(t)$ (with $v > 0$ if the ball is falling downwards) satisfies

$$m\frac{dv}{dt} = mg - kv\,, \qquad v(0) = 0\,,$$

where $k > 0$ is a drag constant.

- (i) Calculate the velocity at any later time t before the ball hits the ground.
- (ii) Give a formula for the time at which the ball hits the ground.