HOMEWORK 2 due in class on September 24th

Problem 1: Use an integrating factor to solve the following initial value problems for $y = y(t)$:
(a) $y' + 4y - e^{-t} = 0$, $y(0) = 4/3$.
(b) $(t^2+1)y' + ty = t\sqrt{t^2+1}$, $y(0) = 0$.
(c) $\cos(t)y' + \sin(t)y = 2t\cos^2(t)$, $y(\pi/4) = -15\sqrt{2}\pi^2/32$.

Problem 2: The secretion of hormones into the blood is often a periodic activity. If a hormone is secreted on a 24-hour cycle, then the rate of change in the level of hormone in the blood may be represented by the initial value problem:

$$\frac{dx}{dt} = \alpha - \beta\cos\left(\frac{\pi t}{12}\right) - kx, \qquad x(0) = x_0,$$

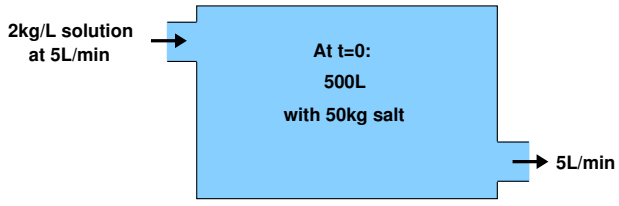
where $x(t)$ is the amount of hormone in the blood at time t, α is the average secretion rate, β is the amount of daily variation in the secretion, and k is a positive constant reflecting the rate at which the body removes hormones from the blood. If $\alpha = \beta = 1$, $k = 2$, and $x_0 = 10$, solve for $x(t)$.

Problem 3: The temperature T (in units of 100°F) of a university classroom on a cold winter day varies with time t (in hours) as:

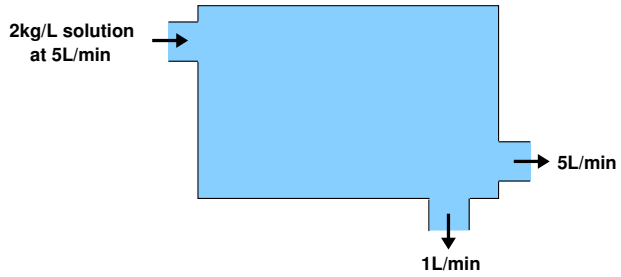
$$\frac{dT}{dt} = \begin{cases} 1 - T, & \text{if the heating unit is ON.} \\ -T, & \text{if the heating unit is OFF.} \end{cases}$$

Suppose that $T = 0$ at 9:00am, the heating unit is ON from 9-10am, OFF from 10-11am, ON again from 11am-noon, and so on for the rest of the day.
(a) How warm will the classroom be at noon? At 5pm?
(b) Plot the solution for $0 < t < 72$ hours.

Problem 4: Suppose a brine containing 2kg of salt per liter runs into a tank initially filled with 500L of water containing 50kg of salt. The brine enters the tank at a rate of 5 L/min. The mixture, kept uniform by stirring, is flowing out at a rate of 5 L/min. See figure (a).



(a) $0 < t \leq 10$min



(b) $t > 10$min

(a) Find the concentration, in kg/L of salt in the tank after 10 min.
Hint: Let A denote the number of kg of salt in the tank at t minutes after the process begins and use the fact that:

rate of increase of A = rate of input - rate of exit.

(b) After 10 min, a leak develops in the tank and an additional liter per minute of mixture flows out of the tank (see figure (b)). What will be the concentration, in kg/L of salt in the tank, 20 min after the leak develops?

Problem 5: Solve the following initial value problems for $y = y(t)$:
(a) $y'' - y = 0$, $y(0) = 4$, $y'(0) = 2$.
(b) $3y'' + 5y' - 2y = 0$, $y(0) = 7$, $y'(0) = 0$.
(c) $y'' + 2y' + 2y = 0$, $y(0) = 0$, $y'(0) = 1$.
(d) $y'' + 6y' + 5y = 0$, $y(0) = 2$, $y'(0) = 2$.