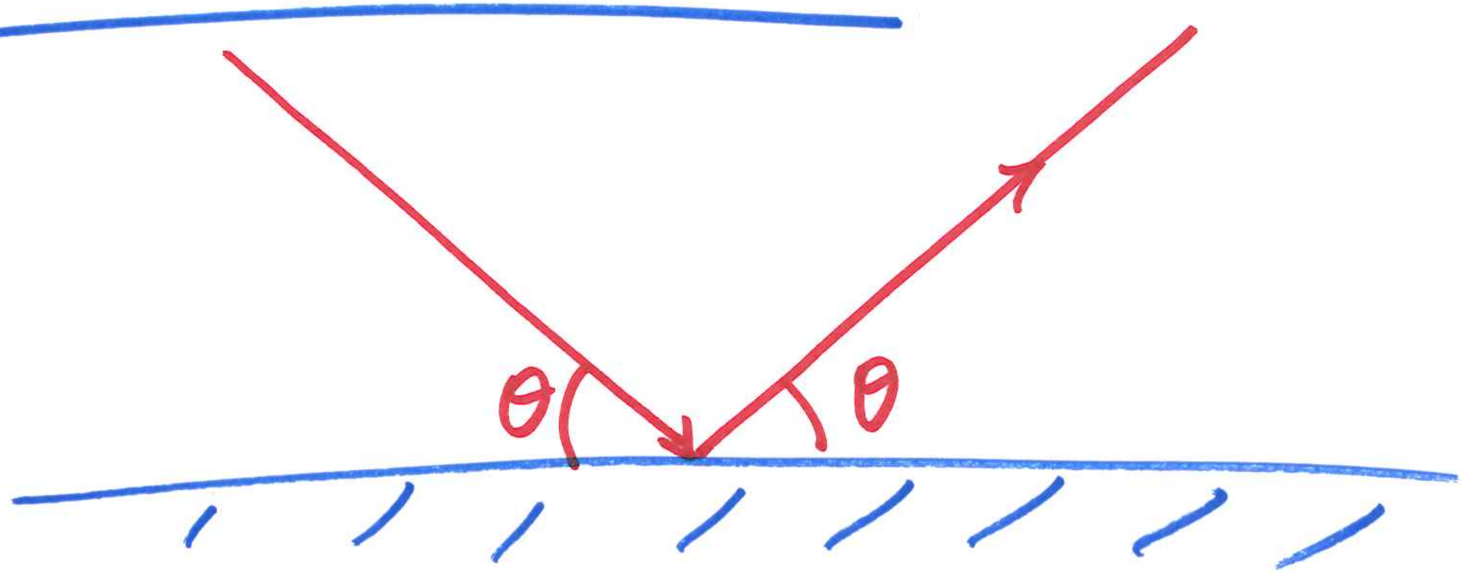
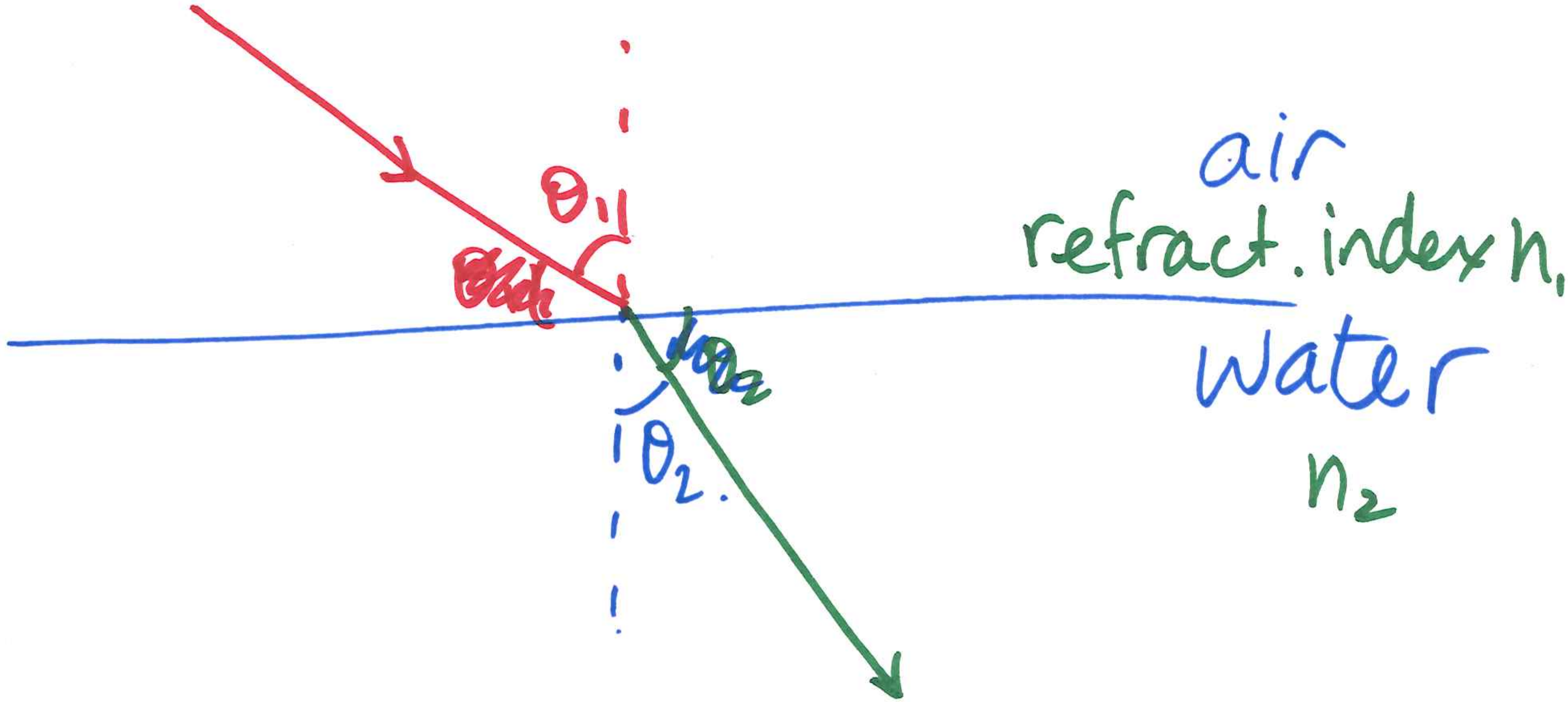
Law of reflection



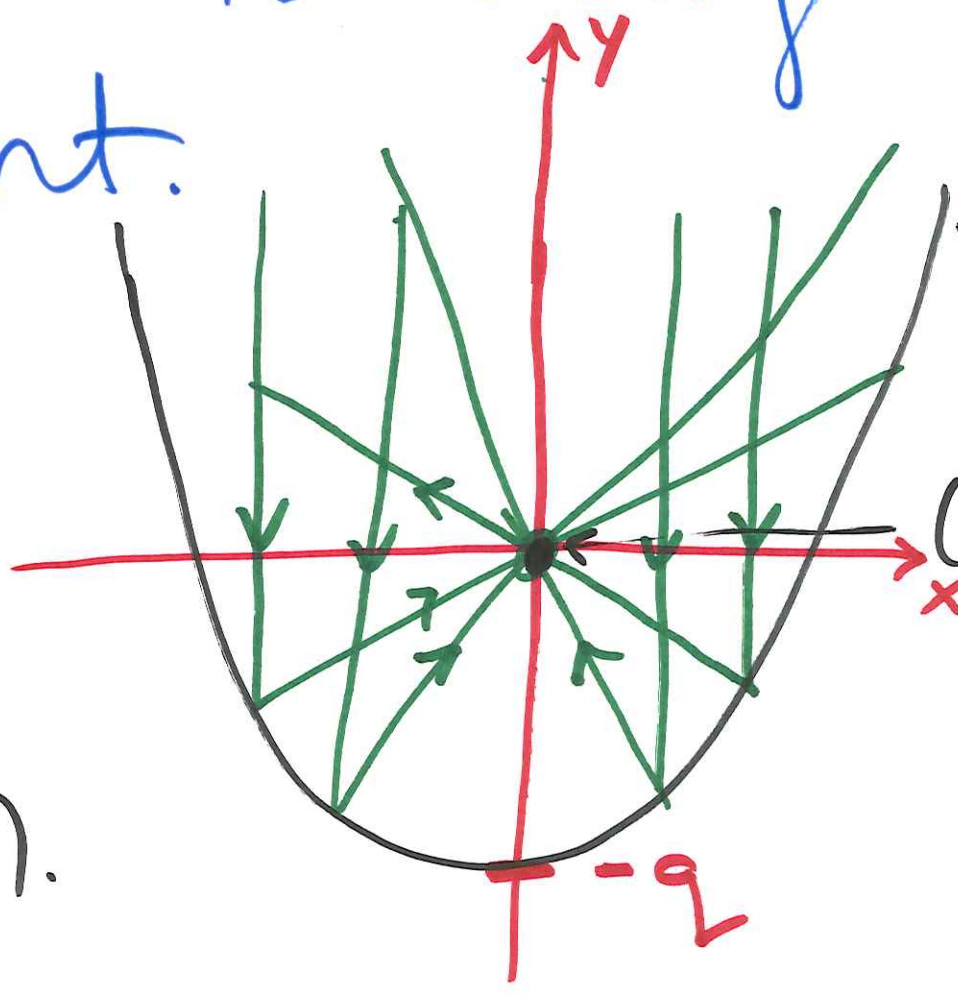
Angle of incidence
= angle of reflection.

Law of refraction (Snell).



$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

Suppose we want to build a mirra that reflects parallel light to a single focal point.

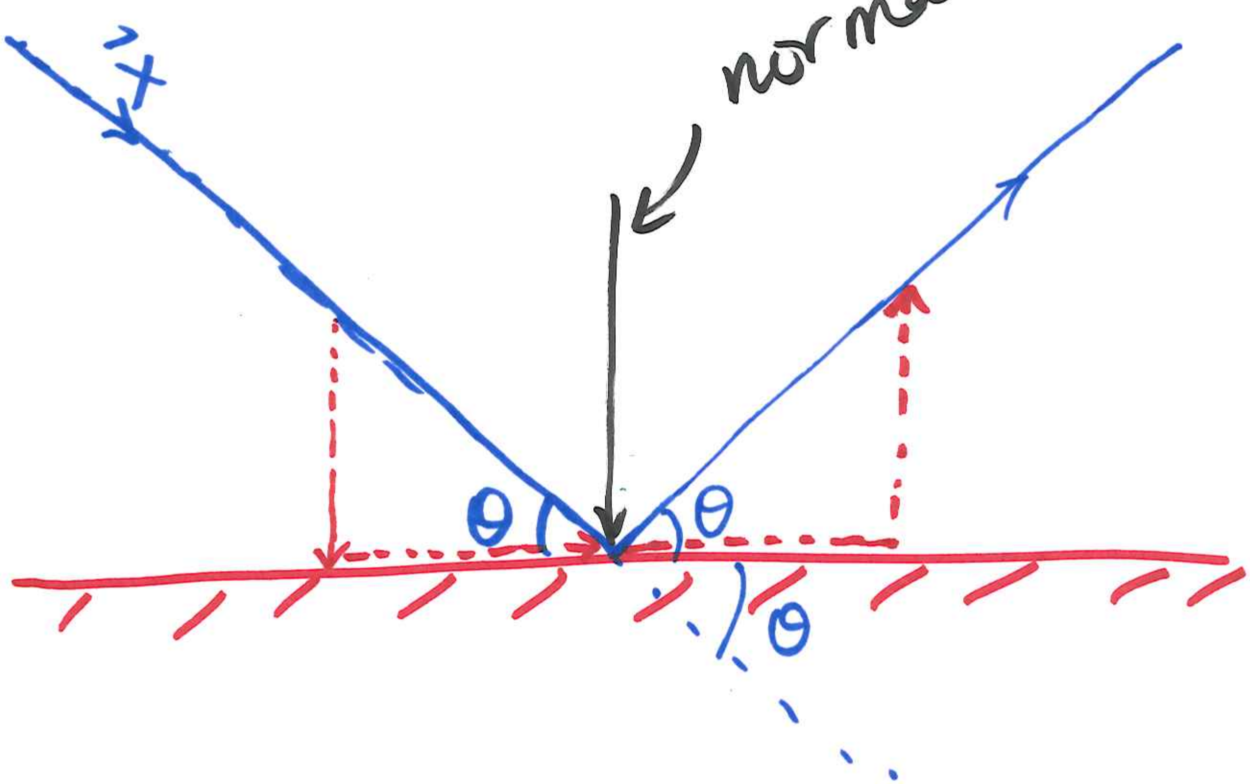


every pt $(x, y(x))$.

$y(0) = -q$

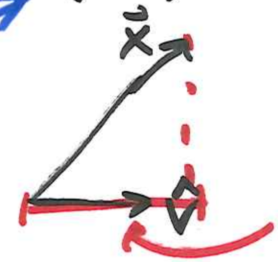
<u>Goal</u>: find $y(x)$.

To get there: what happens to our ray of light as it is reflected?



length of vert. component: $\hat{v}^T \vec{x} = \frac{\vec{v}^T \vec{x}}{\sqrt{\vec{v}^T \vec{v}}}$

unit normal



vertical component with direction:

$$\hat{v}\hat{v}^T \vec{x} = \frac{\vec{v}\vec{v}^T \vec{x}}{\vec{v}^T \vec{v}}$$

Reflection matrix:

identity

$$R = I - 2\frac{\vec{v}\vec{v}^T}{\vec{v}^T \vec{v}}$$

Tangent vector $(1, dy/dx)$ so let's take normal vector $\vec{v} = \left(-\frac{dy}{dx}, 1\right)$.

Take $\vec{x}$ a ray from our focus and bounce it off mirror; want reflected ray to be vertical.

↳ $(x, y(x))$.

horizontal. vertical

$$\begin{bmatrix} 1 & 0 \end{bmatrix} R \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

That's it! The rest is plugging in.

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \left(\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{(y')^2+1} \begin{bmatrix} (y')^2 & -y' \\ -y' & 1 \end{bmatrix}}_{R} \right) \begin{bmatrix} x \\ y \end{bmatrix} = 0.$$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$: $I.$

$\frac{1}{(y')^2+1} \begin{bmatrix} (y')^2 & -y' \\ -y' & 1 \end{bmatrix}$: $-2\frac{\vec{v}\vec{v}^T}{\vec{v}^T\vec{v}}$

$\Rightarrow \dots$

$$\Rightarrow \boxed{x\left(\frac{dy}{dx}\right)^2 - 2y(x)\frac{dy}{dx} - x = 0}$$

MIRROR EQ.