Forced Oscillations Example

Consider the forced LCR circuit, which can be described by:

$$Q''(t) + RQ'(t) + \frac{1}{C}Q = F\cos(\beta t), \quad Q(0) = 0, \quad Q'(0) = 1,$$

where Q(t) is the charge at time t, R the resistance, C the capacitance, and the inductance $L = 1$. The forcing has amplitude F and frequency β.
On the following page you will find plots of different solutions $Q(t)$ for various values of R, C, F, and ω, corresponding in no particular order to:

A $R = 1, C = 1/100, F = 1, \beta = 9 \Rightarrow$
$Q''(t) + Q'(t) + 100Q = \cos(9t), \quad Q(0) = 0, \quad Q'(0) = 1.$

B $R = 0, C = 1/16, F = 1, \beta = 4 \Rightarrow$
$Q''(t) + 16Q = \cos(4t), \quad Q(0) = 0, \quad Q'(0) = 1.$

C $R = 0, C = 1/16, F = 0, \beta$ not relevant $\Rightarrow$
$Q''(t) + 16Q = 0, \quad Q(0) = 0, \quad Q'(0) = 1.$

D $R = 1, C = 1/100, F = 1, \beta = 10 \Rightarrow$
$Q''(t) + Q'(t) + 100Q = \cos(10t), \quad Q(0) = 0, \quad Q'(0) = 1.$

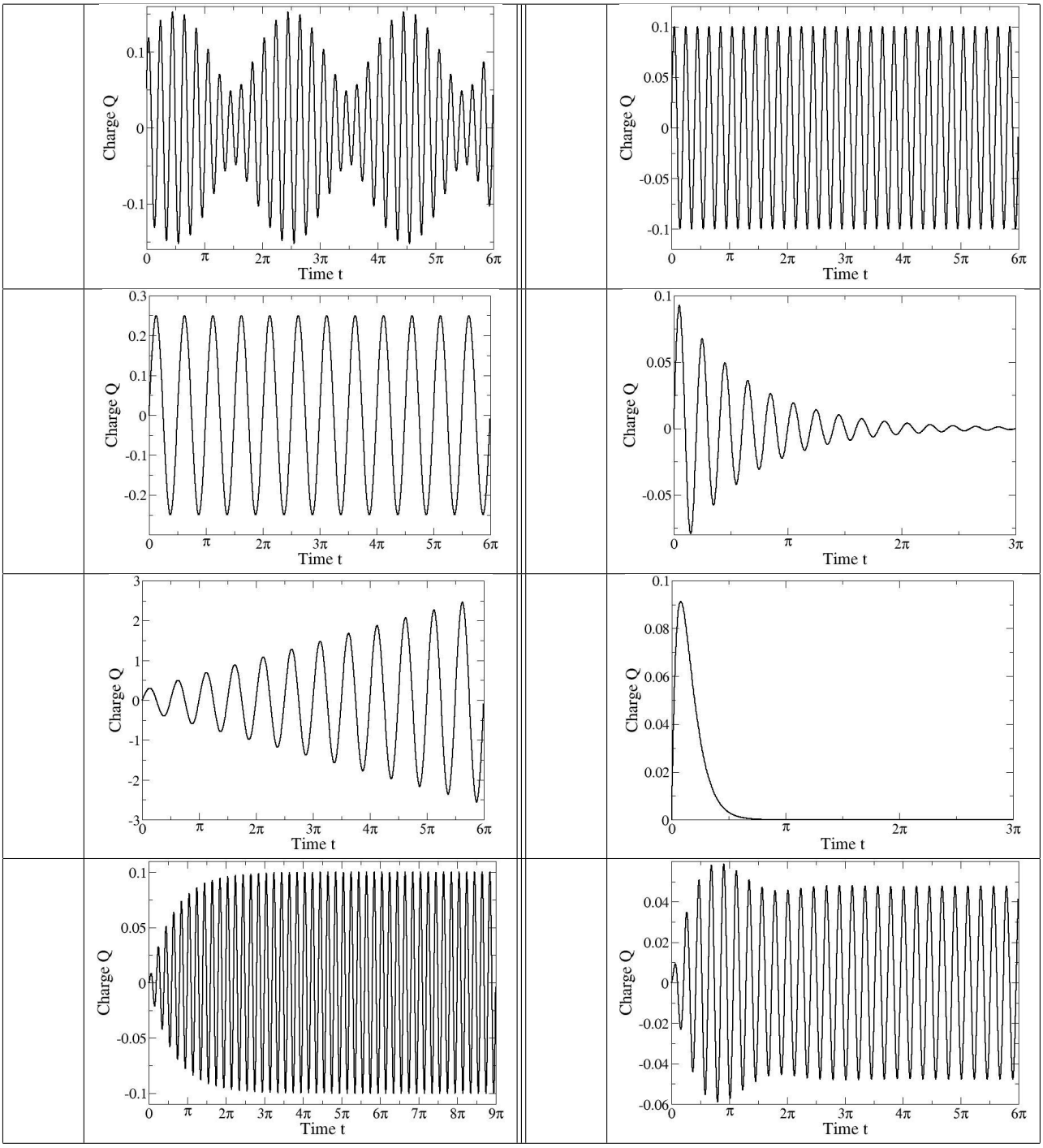
E $R = 1, C = 1/100, F = 0, \beta$ not relevant $\Rightarrow$
$Q''(t) + Q'(t) + 100Q = 0, \quad Q(0) = 0, \quad Q'(0) = 1.$

F $R = 0, C = 1/100, F = 0, \beta$ not relevant $\Rightarrow$
$Q''(t) + 100Q = 0, \quad Q(0) = 0, \quad Q'(0) = 1.$

G $R = 8.1, C = 1/100, F = 0, \beta$ not relevant $\Rightarrow$
$Q''(t) + 8.1Q'(t) + 16Q = 0, \quad Q(0) = 0, \quad Q'(0) = 1.$

H $R = 0, C = 1/100, F = 1, \beta = 9 \Rightarrow$
$Q''(t) + 100Q = \cos(9t), \quad Q(0) = 0, \quad Q'(0) = 1.$

Match the differential equation with its solution's plot by writing the appropriate letter in the box to the *left* of the plot, on the following page.
Hints:
1. You don't need to solve each problem completely.
2. Look at the plots carefully and don't forget to check the axis scales.


Charge Q
Time t
0
π
2π
3π
4π
5π
6π
7π
8π
9π
0.1
0
-0.1
0.05
-0.05
0.3
0.2
-0.2
3
2
1
-1
-2
-3
0.08
0.06
0.04
0.02
-0.02
-0.04
-0.06