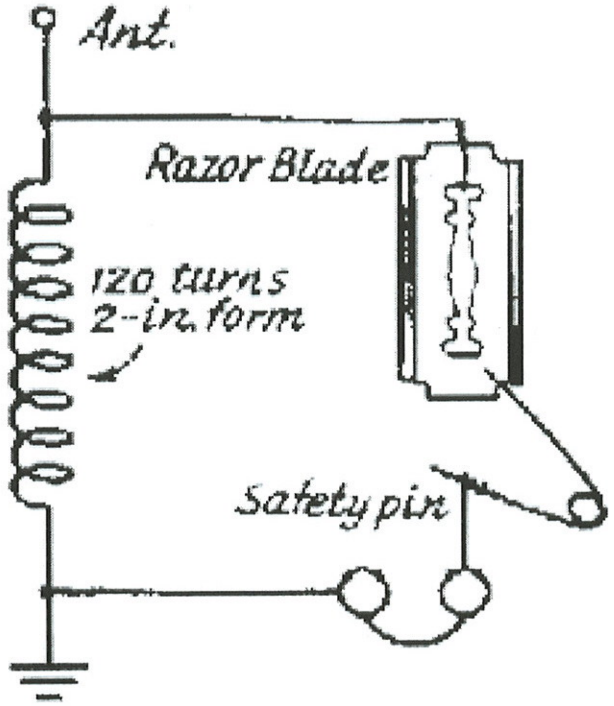
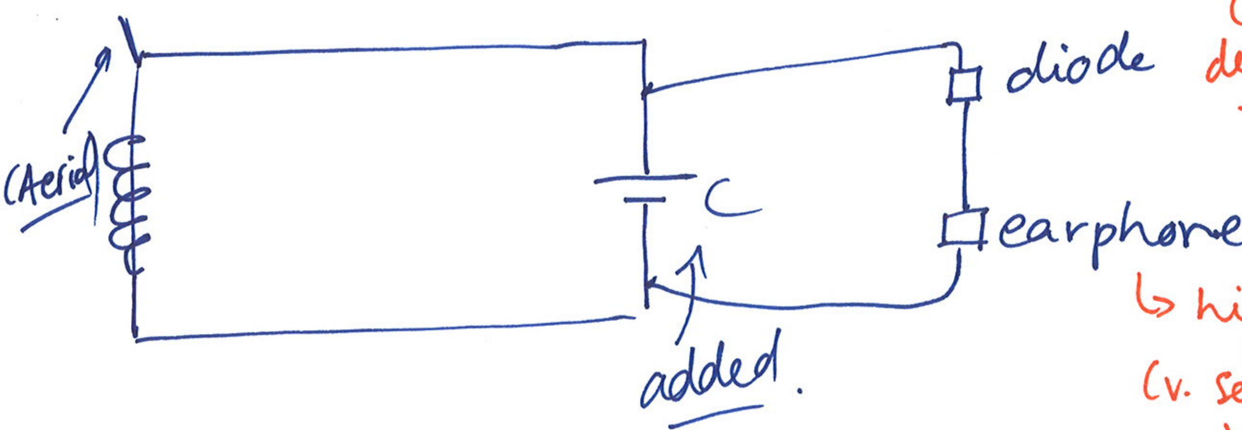
Crystal Radio



Fox Hole Radio
(http://www.wikihow.com/Make–a–Crystal–Radio)



"Crystal" detector, used to be galena.

↳ high-impedence (v. sensitive to voltage change)

$$LQ'' + \frac{1}{C}Q = \underbrace{A\cos\beta t}$$

RF signal, assume ~~sign~~ single-freq.

Solving:

$$Q = \underbrace{C_1 \sin\left(\frac{1}{\sqrt{CL}}t\right) + C_2 \cos\left(\frac{1}{\sqrt{CL}}t\right)}_{\text{homog. sol.}} + \underbrace{\frac{A}{L\left(\frac{1}{CL} - \beta^2\right)} \cos(\beta t)}_{\text{part. sol.}}$$

Let $\omega = \frac{1}{\sqrt{CL}}$ (natural frequency).

<u>Coefficient</u> $\dfrac{A}{L(\omega^2 - \beta^2)}$

- A small
- coeff gets big as $\omega \to \beta$.

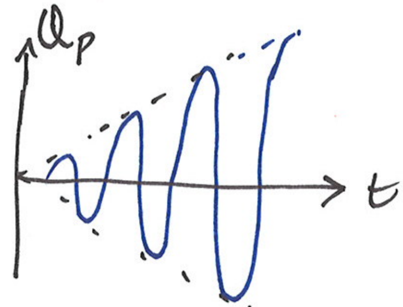
<u>What if $\omega^2 = \beta^2$?</u>

Resonance $\longleftrightarrow$ forcing frequency matches the natural frequency.

ODE: $Q'' + \beta^2 Q = \frac{A}{L}\cos\beta t$ (tuned ω).
(so $\omega = \beta$)

<u>Particular solution:</u>

$$Q_p = \frac{A}{\underset{(\omega=\beta)}{2L\omega}} \cdot t \cdot \cos\beta t.$$



radio explodes ?!?

Why doesn't this happen?

$\longrightarrow$ resistance in wires.

Instead:

$$LQ'' + RQ' + \frac{1}{C}Q = A\cos\beta t.$$

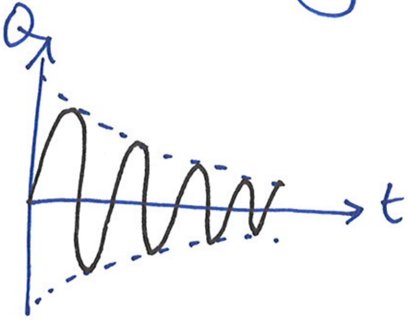
$R > 0$

Assume small resistance, $R < \sqrt{\frac{4L}{C}}$.

(no critical or overdamping).

Homogeneous solution

$\rightarrow$ damped oscillations



(prove for yourself!)

Particular Solution:

$$Q_p = \frac{A\beta R}{L^2\left[(\omega^2-\beta^2)^2+\left(\frac{\beta R}{L}\right)^2\right]}\sin(\beta t)$$

$$+ \frac{A(\omega^2-\beta^2)}{L\left[(\omega^2-\beta^2)^2+\left(\frac{\beta R}{L}\right)^2\right]}\cos(\beta t)$$

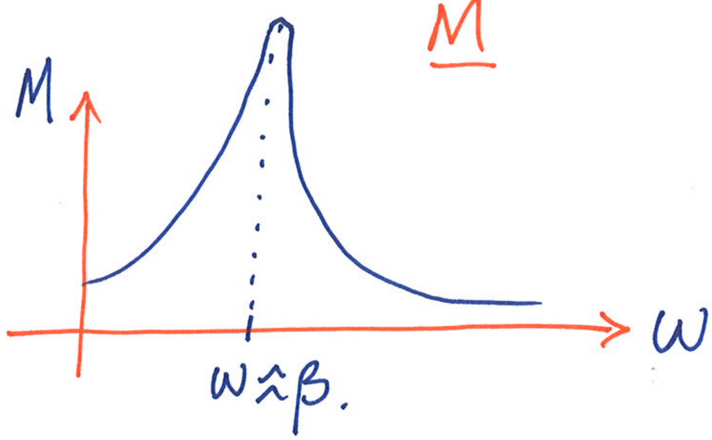
$\omega = \frac{1}{\sqrt{CL}}$ "natural freq."

(undetermined coefs or variation of parameters).

OR

$$Q_p = \underbrace{\frac{A}{L\sqrt{(\omega^2-\beta^2)^2+\left(\frac{\beta R}{L}\right)^2}}}_{\text{amplitude of oscillations}} \cos\left\{\beta t - \tan^{-1}\left(\frac{\beta R}{L(\omega^2-\beta^2)}\right)\right\}$$

<u>M</u>



Amplified only (maximized amplitude of oscs by tuning natural freq. ω to forcing freq. β $\Rightarrow$ exploit RESONANCE)

So what if we have multiple inputs?

$$LQ'' + RQ' + \frac{1}{C}Q = \sum_{n=1}^{N} A_n \cos(\beta_n t).$$

What if I want my radio to pick up info. carried on freq. β_3 only?

$$Q_p = \sum_{n=1}^{N} \frac{A_n}{L\sqrt{(\omega^2-\beta_n^2)^2 + \left(\frac{\beta_n R}{L}\right)^2}} \cos\left\{\beta_n t - \tan^{-1}\left[\frac{\beta_n R}{L(\omega^2-\beta_n^2)}\right]\right\}.$$

want coeff. of $\cos\{\beta_3 t - \tan^{-1}[\ \ ]\}$

to be big.

i.e. $$\frac{A_3}{L\sqrt{(\omega^2-\beta_3^2)^2 + \left(\frac{\beta_3 R}{L}\right)^2}}$$

$\Rightarrow$ ~~Fam~~ Tune radio by changing capacitance so $\omega = \frac{1}{\sqrt{CL}} \approx \beta_3$