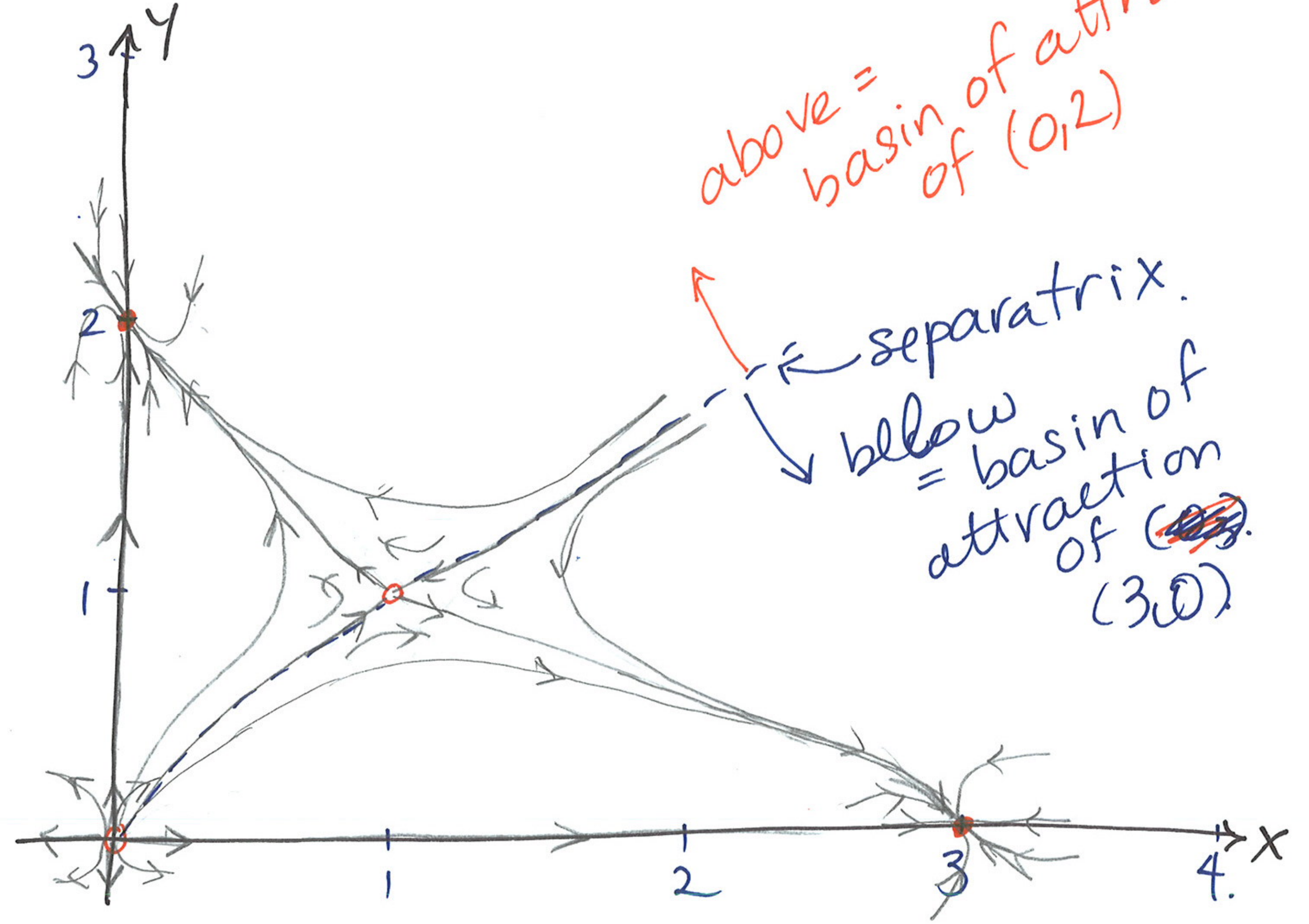
LECTURE 32

From last time:



Linearization results:

$(0,0)$: $r_1 = 3$, $\vec{\xi}_1 = \begin{pmatrix}1\\0\end{pmatrix}$; $r_2 = 2$, $\vec{\xi}_2 = \begin{pmatrix}0\\1\end{pmatrix}$

$(0,2)$: $r_1 = -1$, $\vec{\xi}_1 = \begin{pmatrix}1\\-2\end{pmatrix}$; $r_2 = -2$, $\vec{\xi}_2 = \begin{pmatrix}0\\1\end{pmatrix}$

$(3,0)$: $r_1 = -3$, $\vec{\xi}_1 = \begin{pmatrix}1\\0\end{pmatrix}$; $r_2 = -1$, $\vec{\xi}_2 = \begin{pmatrix}3\\-1\end{pmatrix}$

$(1,1)$: $r_1 = -1+\sqrt{2}$, $\vec{\xi}_1 = \begin{pmatrix}\sqrt{2}\\-1\end{pmatrix}$; $r_2 = -1-\sqrt{2}$, $\vec{\xi}_2 = \begin{pmatrix}\sqrt{2}\\1\end{pmatrix}$.

Notice:

~~On eith~~
2 basic areas where ICs will take us to either (0,2) or (3,0)

"Basins of attraction" of (0,2) and (3,0) respectively

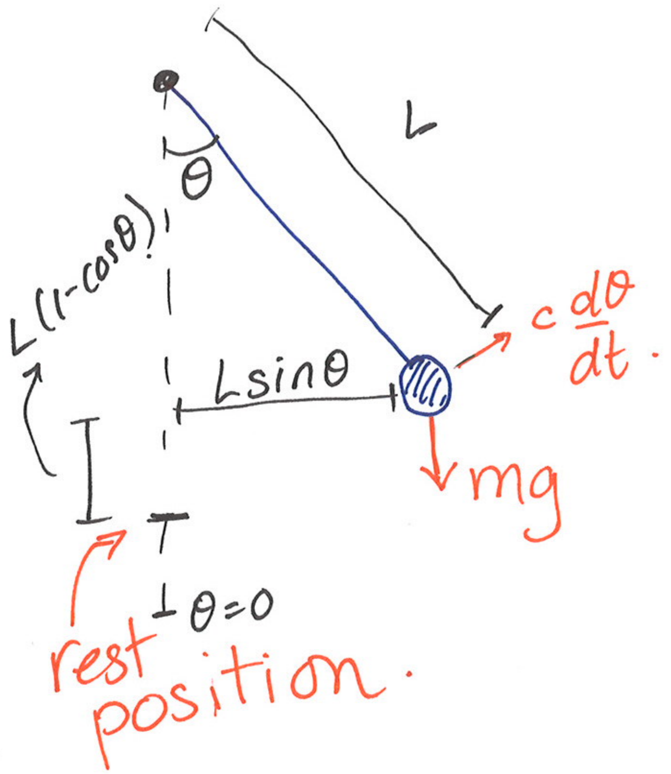
Def'n: The set of all initial points from which trajectories approach a given, asymptotically stable critical point is called the "basin of attraction" or "region

of asymptotic stability"
for that critical point.

⇒ Regions are bounded
by separatrices that
pass through a neighbour-
ing saddle point.

The nonlinear pendulum

→ p.t.o.



Use principle of angular momentum.

Note: torque on mass

$$= \vec{r} \times \vec{F}$$

$$= (L\sin\theta)(mg) \quad \} \text{ sloppy}$$

$$= mgL\sin\theta.$$

Principle:

$$mL^2\frac{d^2\theta}{dt^2} = -cL\frac{d\theta}{dt} - mgL\sin\theta.$$

$$\frac{d^2\theta}{dt^2} = \frac{-c}{mL}\frac{d\theta}{dt} - \frac{g}{L}\sin\theta.$$

let $\gamma = \frac{c}{mL}$, $\omega^2 = \frac{g}{L}$

obtain: $$\boxed{\frac{d^2\theta}{dt^2} + \gamma\frac{d\theta}{dt} + \omega^2 \sin\theta = 0}$$

eq. for pendulum..

May have seen linear pendulum:

$$\frac{d^2\theta}{dt^2} + \gamma\frac{d\theta}{dt} + \omega^2\theta = 0.$$

To obtain: - assume θ small
- $\sin\theta \approx \theta$ (T.S.)

$\therefore$ only good for small θ.

Start by considering undamped pendulum. ($\gamma = 0$):

$$\frac{d^2\theta}{dt^2} + \omega^2 \sin\theta = 0.$$

This is a "conservative system":

$$\dot\theta\ddot\theta + \omega^2(\sin\theta)\dot\theta = 0$$

$$\frac{d}{dt}\left(\frac{\dot\theta^2}{2}\right) - \frac{d}{dt}\left(\omega^2\cos\theta\right) = 0$$

integrating:

$$\frac{\dot\theta^2}{2} - \omega^2\cos\theta = \underset{\text{(const.)}}{C}$$

conserved quantity.

$\longrightarrow$ energy.

to within multiplicative const:

$\dot\theta^2/2 \rightsquigarrow$ kinetic energy

$\omega^2 \cos\theta$ ~> related to potential energy.

moving on...

Write as a 1st order linear system.

Let $x = \theta$, $y = \dot{\theta}$

then $\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -\omega^2 \sin x \end{cases}$

system for NL pendulum w/ <u>no</u> damping.

Find critical points:

where $\dfrac{dx}{dt} = 0$ & $\dfrac{dy}{dt} = 0$ $\longrightarrow$

Critical points
$y = 0$
$x = n\pi$
$(n \in \mathbb{Z})$.
$(\sin x = 0)$

n even → pendulum in down position

n odd → pendulum in up position.