LECTURE 33

Broad overview:

(keywords etc)

① First order equations.

Techniques:

- separable eqs
- integrating factors
- homogeneous eqs ($v = y/x$ sub)
- direction fields.

Concepts:

- nonlinear vs linear
- autonomous eqs
- existence & uniqueness.

② Second order linear eqs:

Techniques:

- homog. eq → e^{rt} sub which gives the char. eq.
  (roots: real, complex, distinct, repeated, each w/ a different sol. type)
- reduction of order
- for particular sol:
  - → Variation of params
  - → Undetermined coefs (guessing wisely).

- solving IVPs
  (a) homog sol.
  (b) part. sol
  (c) general = homog + part.
  (d) ONLY THEN solve for constants using ICs.
- Wronskian.

Concepts:
- Wronskian & linear independence.
- (existence + uniqueness)
- applications to mechanical vibrations.
  → mass - spring
  → LCR.
- resonance, beats, etc.
  (Forcing vs natural freq.)

- (sketches of solutions) ☆

(3) Laplace Transforms.

Techniques:

- direct computation of transform from integral.
- solving IVPs via Laplace transforms (using tables, in particular for inverting transform)
- convolutions

Concepts:

- Region of validity ($s > a$ etc.) ↑

- exponential order / piecewise continuity (for transform to exist).

- step functions ✱ (disc. solutions)
- impulse functions.

~~resu~~ continuity of resulting solution.

④ Linear Systems.

Techniques:

- homog. eq.
  → solve via e-vals / e-vects.

- inhomog. eq.
  → Variation of Vectors
  → Undet Coefs (Vectors).

→ diagonalization.

- IVPs
- phase plane eq.
- phase portraits.

linear ↳ homog. syst. only. here.

Concepts:

- analogy to 2nd order linear eq
  → this is a generalization.

⑤ Nonlinear systems.

Techniques:

- Finding critical points.
- Linearize → Jacobian.
- Classify type/stability

of critical points
(by analysing local
linear system.)
(linearize about crit.
point., evals of
resulting matrix).

- phase portraits
  sketching ★
  (use eigenvectors NEAR
  RELATED CRIT POINT ONLY,
  nullclines, etc).

Concepts:

- critical point & type/
  stability
- almost linear systems.